Implied Volatility and Predictability of GARCH Models

Vivek Rajvanshi[1], Arijit Santra[2], and Saunak Basu[3]

: We have examined the predictive power of GARCH model to forecast return volatility for Nifty 50 index. Realized volatility, which is the sum of intraday squared returns, is used as the proxy for the true volatility. Three models of the GARCH family have been used to forecast return volatility i.e., GARCH, GJR-GARCH and EGARCH along with their implied volatility (IV) augmented counterparts i.e., GARCH IV, GJR-GARCH IV and EGARCH IV. Implied Volatility forecasting has been done using AR, MA, ARMA, ARIMA and Random Walk. But GARCH model augmented with implied volatility perform better than GARCH models without augmentation or implied volatility alone. Forecasting performance of the competing models is judged by using mean absolute error (MAE) and root mean squared error (RMSE). MAE and RMSE show that GARCH IV model is best suited for the volatility forecasting in the context of Nifty 50 index.

[1] Correspoore1

INTRODUCTION

The return process follows the mean equation:

$r_t = \mu + \varepsilon_t$

where μ is the constant mean and $\varepsilon_t = h_t z_t$ is the innovation with $z_t \sim N(0,1)$

The variance equation: $h_t^2 = \alpha_0 + \alpha_1 \varepsilon^2_{t-1} + \beta_1 h^2_{t-1}$ (1)

GARCH model has been very successful in estimating and forecasting return volatility and capturing the stylized facts, such as long memory, of return volatility.

With IV augmented, the equation becomes: $h_t^2 = \alpha_0 + \alpha_1 \varepsilon^2_{t-1} + \beta_1 h^2_{t-1} + \theta\, IV^2_{t-1}$ (2)

GJR-GARCH model, proposed by Glosten, Jagannathan, and Runkle (1993), takes into account the leverage effect along with long memory.

The variance equation:

$h_t^2 = \alpha_0 + \alpha_1 \varepsilon^2_{t-1} + \beta_1 h^2_{t-1} + \gamma\, \varepsilon^2_{t-1} I_{t-1}$ (3)

With IV augmented, the variance equation becomes

$h_t^2 = \alpha_0 + \alpha_1 \varepsilon^2_{t-1} + \beta_1 h^2_{t-1} + \gamma\, \varepsilon^2_{t-1} I_{t-1} + \theta\, IV^2_{t-1}$ (4)

Here, the leverage effect is captured by γ, such that, $I_{t-1} = 1$ if $\varepsilon_{t-1} < 0$ and $I_{t-1} = 0$ if $\varepsilon_{t-1} > 0$.

EGARCH model, proposed by Nelson (1991) captures the leverage effect as well with the long memory property of the return volatility.

The variance equation:

$\ln(h_t^2) = \alpha_0 + \alpha_1 \mid \varepsilon_{t-1} \mid / h_{t-1} + \gamma\, \varepsilon_{t-1} / h_{t-1} + \beta_1 \ln(h^2_{t-1})$ (5)

With IV augmented, the equation becomes

$\ln(h_t^2) = \alpha_0 + \alpha_1 \mid \varepsilon_{t-1} \mid / h_{t-1} + \gamma\, \varepsilon_{t-1} / h_{t-1} + \beta_1 \ln(h^2_{t-1}) + \theta\, IV^2_{t-1}$ (6)

Where the coefficient γ captures the presence of the leverage effects if $\gamma < 0$.

Here we want to forecast the actual volatility using implied volatility and realized volatility. In order to investigate whether the IV index model forecast or RV forecast will be more accurate than the GARCH type models, AR, MA, ARMA, ARIMA and Random Walk models are going to be used.

India VIX is a volatility index computed by NSE based on the order book of NIFTY Options. For this, the best bid-ask quotes of near and next-month NIFTY options contracts which are traded on the F&O segment of NSE are used. India VIX indicates the investor's perception of the market's volatility in the near term i.e. it depicts the expected market volatility over the next 30 calendar days.

Daily implied volatility is obtained from VIX index using the formula VIX/100/sqrt(250).

Realized Variance is the sum of 5-minute intraday squared returns. It is calculated using the formula $\sigma_t^2 = \Sigma\, r^2_{t,j}$ where $r_{t,j}$ is the return in interval j on day t

Autocorrelation	Partial Correlation
|*** |	|*** |
|** |	|* |
|** |	|* |
|** |	|* |
|** |	|* |
|** |	|* |
|** |	| |
|** |	| |
|** |	| |
|** |	| |

From the correlogram test, we observe that both implied volatility and realized volatility series have a large partial correlation at AR(1).

So, for each IV and RV series, AR(1), MA(1), ARMA(1,1) and ARIMA(1,1,1) models are used.

The mean equations

A generalization of the ARMA models is the autoregressive integrated moving average (ARIMA) model. It is usually denoted as ARIMA (p, d, q) and is employed to capture the possible presence of short memory features in the dynamics of implied volatility. The ARIMA (1,1,1) specification is given by

$$\Delta IV_t = c_0 + \varphi_1 \Delta IV_{t-1} + \theta_1 \varepsilon_{t-1} + \varepsilon_t \quad (13)$$

$$\Delta RV_t = c_0 + \varphi_1 \Delta RV_{t-1} + \theta_1 \varepsilon_{t-1} + \varepsilon_t \quad (14)$$

$$IV_t = IV_{t-1} + \varepsilon_t \quad (15)$$

$$RV_t = RV_{t-1} + \varepsilon_t \quad (16)$$

IN-SAMPLE RESULTS

As mentioned, the in-sample period is from 1st January 2012 to 31st December 2014.

	GARCH	GJR GARCH	EGARCH
α_0	1.40E-06 (0.0485)	1.94E-06 (0.0046)	-0.203765 (0.0017)
α_1	0.038811 (0.0011)	-0.004671 (0.6763)	0.069476 (0.0011)
β_1	0.944393 (0.0000)	0.942231 (0.0000)	0.983968 (0.0000)
γ		0.080098 (0.0001)	-0.066159 (0.0000)
Log-Likelihood	2422.352	2430.868	2428.474

(Values in brackets indicate the p-values)

The constant term α_0 is statistically significant at the 5% level for all the three GARCH specifications. α_1, β_1 and γ are statistically significant at the 1% level except for α_1 in GJR GARCH.

As depicted in table above ARMA model is seems to be best predictor when we are using realized volatility to forecast.

Overall, among GARCH volatility, implied volatility and realized volatility, realized volatility, the best predictor is realized volatility using ARMA Model.

In the following diagram, we have chosen the best method from each of GARCH volatility, implied volatility and realized volatility and compared the forecasts in the out-of-sample period with the actual volatility.

CONCLUSION

This paper provides a comparative evaluation of the ability of a range of GARCH, IV and RV models to forecast the Nifty 50 return volatility. A total of six GARCH models have been considered, i.e., GARCH, GJR GARCH, EGARCH, GARCH IV, GJR GARCH IV, EGARCH IV. Additionally, AR, MA, ARMA, ARMA and Random Walk Models have been used for forecasting with implied volatility and realized volatility.

ARIMA performs the best when we are analysing the forecasting ability of IV. In case of RV, ARMA performs the best. As for the GARCH models, the inclusion of IV in the GARCH variance equations improves the out-of-sample performance of the GARCH models.

REFERENCES

Andersen, T. G. (1998). Answering the skeptics: Yes, standard volatili