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The Price-Setting Newsvendor Model with Variable Salvage Value

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Retailers of short product life cycle items, like apparel and fast-moving consumer goods, face the challenges of uncertainty and obsolescence. The retailer stocks up products at the beginning of the selling period, with little information about what the demand would be. If the quantity procured turns out to be different from the actual demand observed, retailer faces either leftovers or stock

distribution of

Theorem 1 identifies the conditions for which the optimal solution for a single period variable salvage value newsvendor problem can be indentified analytically. In the following section we analyze the optimal condition(s) for multiplicative demand.

2.2. Multiplicative Demand Scenario

In the case of multiplicative demand the order quantity (q) is defined as, $q \quad d(p).z$ where $y(p) \quad ap^{\ b}$ $(a \quad 0, b \quad 1)$. The leftover quantity is expressed by the following equation: $I \quad q \quad d(p, \ ) \quad (z \quad) .d(p)$.The salvage value is assumed to be a multiplicative function of the left over inventory and is given by, $v(I) \quad a_v I^{\ b_v} \ 0 \quad b_v \quad 1$. In order to obtain closed form solutions, we further assume, $b_v \quad \frac{1}{b}$ without where, $D(p) \quad (p \quad c)\, y(p)$ and (z) is previously defined. In.

conditions under which the former performs superiorly. For the purpose of brevity we restrict the comparison to the additive demand case.

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As previously mentioned in section 1, Cachon and Kök [1] have proposed different heuristics for estimating the fixed salvage value. They have demonstrated that the weighted average salvage value (WASV) heuristic provides the solution that is closest to the optimal one. The WASV heuristic computes the fixed salvage value by the relationship $v(q_C) = R_s(q_C)/I(q_C)$, where q_C represents the classical newsvendor optimal order quantity. At the classical newsvendor optimal order quantity, $v(q_C) = p - (p - c)/F(q_C)$. In this section, we evaluate the variable salvage value newsvendor model against the classical newsvendor model where salvage value is computed using the WASV heuristic.

? AA4! ><! The optimal profit in the variable salvage value newsvendor model will at least be equal to the optimal profit obtained for the classical newsvendor in the case of additive demand.!

"#00@<! For additive demand, the fixed salvage value as per WASV heuristic is represented by $v(q_C) = R_s(q_C)/I(q_C)$ where $q_C = y(p) + z_C$. $y(p)$ is the deterministic part of the demand and z_C is the excess order quantity computed according to classical newsvendor model. The classical newsvendor profit function (Π_C) and variable salvage value newsvendor profit function (Π_{VSP}) are given by,

$$\Pi_C(p,v,q) = E[R_1(p,q)] + E[R_s(v,q)] - cq \qquad \text{... (13)}$$

$$\Pi_{VSP}(p,a_v,b_v,q) = E[R_1(p,q)] + E[R_s(a_v,b_v,q)] - cq \qquad \text{... (14)}$$

For a given price p, at $q = q_C$ the clearance revenue yield through both the models would be same, We subsequently prove in Lemma 3 that variable salvage value model also results in improved quantity decision compared to a classical newsvendor model.!

? A A4B8! If the stochastic component of demand follows the relationship SD( ) 1 in the case of additive demand, then the optimal order quantity, q^*, for a variable salvage value newsvendor is lesser than the classical newsvendor optimal order quantity, q_C, where $/z^*$ represents a truncated distribution of over $[0, z^*]$.

"#00@8 See the appendix.

Through Lemma 2 we prove that the newsvendor model with fixed salvage value results in profit loss compared to a newsvendor model where salvage Md א† value lu) ḓ

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"#00@! 0@!   0# A! ;<! From the first and second order derivative of E (z, p) we establish the convexity of the profit function over the entire range 0,   .

$$/ \qquad {}_v \quad z \quad b_v \int^z F \; u \; du$$

Under the condition $(p \quad a_v)\{1 \quad F(\hat{I})\} \quad (a_v \quad c) \quad 0$, (0) and $(\hat{I})$ are opposite in sign. Then the optimal z^* lies in the range $[0, \hat{I}]$ and it satisfies the condition,

$$(p \quad c) \quad (p \quad a_v)F(z) \quad 2b_v \int_0^z F(u)du \quad 0 \qquad \text{... (A5)}$$

The value of (z) at the end points of the range $(\hat{I}, \quad]$ are given by,

$$(\hat{I} \quad) \quad (p \quad c) \quad (p \quad a_v)F(\hat{I}) \quad 2b_v \int^z F(u)du$$

$B(z, p)/ \quad p \quad (b \quad 1) \quad (z) \quad a_v a^{b_v} \quad 0$

Therefore, at $p^* \quad \frac{(b \quad 1) \; p^0 \quad bc \quad (z) \quad (z)}{(b \quad 1) \quad (z) \quad a_v a^{b_v}}$, $B(z, p) \quad 0$. From (A7) and (A8) it is evident that $E \quad z, p$ is maximum at, $p \quad p^*$.

"#00@!0@! 0# A!><!At $p \quad p^* \quad p(z)$ the first order derivative of the expected profit function

By changing the order of integration, for $z \quad [0, \hat{I}]$, (A14) can be rewritten as:

$$\frac{}{z}\ \frac{R_s(\ )}{I(z)} \qquad \frac{2b_v\ F(z)^{2}}{\left(\int_0^z F(u)du\right)^{2}}\ A_1 A_2 \quad \frac{1}{2}A_3$$

where $A_1 \quad \int_0^z uf(u)du \Big/ F(z)$, $A_2 \quad \int_0^z (z \quad u)\, f(u)du \Big/ F(z)$ and $A_3 \quad \int_0^z z^2 \quad u^2\ f(u)du \Big/ F(z)$. If Y_z represents a random variable that corresponds to the truncated distribution of u over [0, z], then by defining $\quad (Y_z/z)$ we have, $A_1 \quad zE(\ )$, $A_2 \quad zE(1 \quad)$ and $A_3 \quad z^2E(1 \quad {}^2)$.

References

[1] G. P. Cachon, A. G. Kök, How to (and how not to) estimate the salvage value in the newsvendor model, Manufacturing & Service Operations Management 9 (3) (2007) 276-290.

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