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Abstract

For certain types of goods, the multiple unit auctions have to be conducted sequentially. One probable reason for this is that the di erent units of the goods are not available together for putting up for sale. This might happen when the objects are available in batches to the auctioneer and do not come together. In such sequential

Jensen (2007), the retail markets may be (and are quite likely to be) located at different distances from the wholesale market. The larger the distance of a retail market from a wholesale market, the higher is the cost of commuting to that retail market. The bidders who win first would therefore obviously try to capture the nearest markets, while

2. There are two identical indivisible objects up for sale in two stages, with one unit being sold in each stage.

3. Each bidder has demand for a single unit, so that after the first stage the winning bidder of that stage exits.

4. The value that each bidder attaches to one of the objects is common for everyone, denoted by V and it is common knowledge. Before the beginning of the second stage, the winning bid of the first stage is disclosed.

5. There is waiting cost for each individual bidder i denoted by c_i and it is a private information to bidder i.

6. c_i-s are distributed independently and identically over the interval $[\underline{c}; \bar{c}]$ following the same continuous distribution function $F(\cdot)$ with density $f(\cdot)$ and has full support.

7. Thus in the second stage the value for the remaining object for bidder i is $V - c_i$. We denote this net value by x_i.

8. The distribution function of the x_i-s is denoted by the continuous function $G(\cdot)$, with the corresponding density being $g(\cdot)$

the winning type in the rst stage auction is x_1

$$ {}_l(z;x) = (V \quad {}_l(z))\left\{(1 \quad G(z))^{n \quad 1} + (n \quad 1) \int_{\underline{x}}^{x} G(y)^{n \quad 2} dy\right\} G(z) $$

From the rst order conditions of maximisation and in a symmetric equilibrium, we obtain

$$ (V \quad {}_l(x))\, d\left[(1 \quad G(x))^{n \quad 1}\right] \quad {}_l^{=}(x)\left\{(1 \quad G(x))^{n \quad 1} + (n \quad 1) \int_{\underline{x}}^{x} G(y)^{n \quad 2} dy\right\} g(x) = 0 $$

$$) \quad d\left[{}_l(x)(1 \quad G(x))^{n \quad 1}\right] = V d\left[(1 \quad G(x))^{n \quad 1}\right] + (n \quad 1)\left\{\int_{\underline{x}}^{x} G(y)^{n \quad 2} dy\right\} g(x) $$

$$) \quad {}_l(x)(1 \quad G(x))^{n \quad 1} = $$

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Appendix

A.1

The distribution of the lowest order statistic X_1 for $(n - 1)$ values of the random variable X (distributed according to the probability distribution function $G(:)$) is given by

$$G_1(x) = 1 - (1 - G(x))^{n-1}$$

From this we can calculate that the probability for all the $(n - 2)$ bidders' types being higher than the lowest type x_1, is

$$1 - G_1(x_1) = (1 - G(x_1))^{n-2}$$

and analogously the probability for all the $(n-1)$ bidders' types being less z, is

$$(1-G(z))^{n-1}$$

A.2

We can see that

$$\bar{l}'(x) = \frac{(n-1)g(x)}{(1-G(x))^{n-1}}\left[\int_{\underline{x}}^{x} G(y)^{n-2}dy - \frac{(n-1)}{(1-G(x))}\int_{x}^{\bar{x}}\left\{\int_{\underline{x}}^{w} G(y)^{n-2}dy\right\} g(w)\,dw\right]$$
$$= \frac{(n-1)g(x)}{(1-G(x))^{n}}\left[(1-G(x))\int_{\underline{x}}^{x} G(y)^{n-2}dy - (n-1)\int_{x}^{\bar{x}}\left\{\int_{\underline{x}}^{w} G(y)^{n-2}dy\right\} g(w)\,dw\right]$$

Now,

$$\int_{x}^{\bar{x}}\left(\int_{\underline{x}}^{w} G(y)^{n-2}dy \int g(w)\,dw\right)$$
$$= \left[\int_{\underline{x}}^{w} G(y)^{n-2}dy \int g(w)\,dw\right]_{x}^{\bar{x}} - \int_{x}^{\bar{x}}\left\{G(w)^{n-2}\int g(w)\,dw\right\}dw$$
$$= \left[G(w)\int_{\underline{x}}^{w} G(y)^{n-2}dy\right]_{x}^{\bar{x}} - \int_{x}^{\bar{x}}\left\{G(w)^{n-2}G(w)\right\}dw$$
$$= G(\bar{x})\int_{\underline{x}}^{\bar{x}} G(y)^{n-2}dy - G(x)\int_{\underline{x}}^{x} G(y)^{n-2}dy - \int_{x}^{\bar{x}}\left\{G(y)^{n-1}\right\}dy$$
$$= \int_{\underline{x}}^{\bar{x}} G(y)^{n-2}dy - G(x)\int_{\underline{x}}^{x} G(y)^{n-2}dy - \int_{x}^{\bar{x}}\left\{G(y)^{n-1}\right\}dy$$

We denote, $\psi(x) = (1-G(x))\int_{\underline{x}}^{x} G(y)^{n-2}dy - (n-1)\int_{x}^{\bar{x}}\left\{\int_{\underline{x}}^{w} G(y)^{n-2}dy\right\} g(w)\,dw$

We can rewrite this as

$$\psi(x) = (1-G(x))\int_{\underline{x}}^{x} G(y)^{n-2}dy - (n-1)\left[\int_{\underline{x}}^{\bar{x}} G(y)^{n-2}dy - G(x)\int_{\underline{x}}^{x} G(y)^{n-2}dy - \int_{x}^{\bar{x}}\left\{G(y)^{n-1}\right\}dy\right]$$
$$= \int_{\underline{x}}^{x} G(y)^{n-2}dy - G(x)\int_{\underline{x}}^{x} G(y)^{n-2}dy - (n-1)\int_{\underline{x}}^{\bar{x}} G(y)^{n-2}dy + (n-1)G(x)\int_{\underline{x}}^{x} G(y)^{n-2}dy + (n-1)\int_{x}^{\bar{x}}\left\{G(y)^{n-1}\right\}dy$$
$$= \int_{\underline{x}}^{x} G(y)^{n-2}dy + (n-2)G(x)\int_{\underline{x}}^{x} G(y)^{n-2}dy + (n-1)\int_{x}^{\bar{x}}\left\{G(y)^{n-1}\right\}dy - (n-1)\int_{x}^{\bar{x}}\Big\}\,dy \quad n$$

A.3

The expected payoff function, after substituting for the bid function in the first stage, can be written as

$$\pi_1(z;x) = (1 - G(z))^{n-1}V - V + \frac{n-1}{(1-G(z))^{n-1}} \int_z^{\bar{x}} \left\{ \int_{\underline{x}}^{w} G(y)^{n-2}dy \right\} g(w)\,dw$$

$$+ (n-1)G(z)\int_{\underline{x}}^{x} G(y)^{n-2}dy$$

$$= (n-1)\int_z^{\bar{x}} \left\{ \int_{\underline{x}}^{w} G(y)^{n-2}dy \right\} g(w)\,dw + (n-1)G(z)\int_{\underline{x}}^{x} G(y)^{n-2}dy$$

G ((z)dy

$$= (n-1)\left[\int_z^{\bar{x}} \left\{ \int_{\underline{x}}^{w} G(y)^{n-2}dy \right\} g(w)\,dw + G(z)\int_{\underline{x}}^{x} G(y)^{n-2}dy\right]$$

n 2 dy

Now from the first order condition for profit maximisation, we obtain,

$$\frac{\partial \pi_1}{\partial z} = 0$$

$$\Rightarrow (n-1)\left[-\left\{ \int_{\underline{x}}^{z} G(y)^{n-2}dy \right\} g(z) + g(z)\int_{\underline{x}}^{x} G(y)^{n-2}dy \right] = 0$$

$$\Rightarrow g(z)dy$$