INDIAN INSTITUTE OF MANAGEMENT CALCUTTA

WORKING PAPER SERIES

WPS No.719 / December 2012

Pricing Strategies for Gaming-on-Demand

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gaming consoles to play a game. At the same time, gamers get the option to pay according to their usage instead of being compelled to purchase a game. The advantages of this model for the gaming companies are primarily threefold. First, the company can upgrade its games without worrying about the hardware compatibility of its customers. Second, it expands the market to include gamers who are ready to pay a small usage based fee to access online games, but cannot afford to make large upfront payments to buy expensive ones. Third, this model opens up a new revenue stream for outmoded games which are difficult to sell; there may be gamers who will pay to play such a game for a few hours. As the marginal cost of offering a game on demand is very low, it is indeed a profitable model for the gaming company. Moreover, the gaming-on-demand model can help in reducing piracy.

The gaming-on-demand market has already become fiercely competitive. In January 2012, Gaikai Inc., which started gaming-on-demand in 2010, announced a strategic partnership with LG Electronics to launch an integrated Smart TV cloud gaming service [1]. This deal allows LG to leverage the cloud platform of Gaikai to offer a broad range of games to its customers. Games are offered through the game portal service operating within the LG Smart TV. OnLive Inc., Gaikai's main competitor, reacted to this strategic alliance by demonstrating their OnLive Game Service on the next generation LG Smart TV with Google TV (G2 series) in the month of June, 2012 [2, 7]. A few days later Gaikai announced its partnership with Samsung which will offer cloud gaming using Samsung Smart TVs [3]. As competitor rivalries propel the industry towards a greater state of flux, traditional gaming companies will be forced to respond to this challenge emanating from the gaming-on-demand model. One other factor that is expected to favor the shift towards cloud gaming is the increasing popularity of tablets and smart phones [4]. Reports indicate that tablets will become the most important computing device in the future. As more and more people start accessing the Internet using devices with low computing power, gaming companies will increasingly feel the need to offer games that can be played using such devices, in effect, offer games as a service. The recent acquisition of Gaikai by Sony, signifies that this transformation is not just inevitable, it is also imminent [20].

However, there is an obstacle to wide scale adoption of cloud gaming or gaming-on-demand - the quality of broadband services. As the game is delivered as a service via the Internet, the user experience of gaming-on-demand will definitely be poorer if she does not have access to high quality broadband service. Sony, according to reports, seriously considered the idea of coming up with a download only version of its gaming console, Play Station 4. It ultimately rejected that idea keeping in mind the inconsistencies in broadband speeds around the world

Ju Liu [13] developed a framework to study pricing strategies under network e ects, consumer heterogeneity and oligopolistic competition. The work discusses alternate strategies of video games console manufacturers through policy simulations, and critically examines the pricing strategies of Nintendo. The paper reports the ndings of an empirical investigation on console pricing and studies the e ect of vertical integration on the sales performance of video games. Multiple research ndings which indicate that vertical integration leads to lower prices and increased competition in the case of video games console. However, this leads to an increase in the number of units of consoles sold, and a higher demand for video games which leads to higher pro ts [8]. A study by Gill and Warzynski [11] nds that vertically integrated games sell more and at higher prices compared to non-vertically integrated games, which validates the ndings of Derdenger [8]. However, results indicate that there is no e ect of vertical integration on the quality of the games. The problem of optimal pricing over time for a rm selling a durable good to forward looking consumers [15] has been studied in the context of the video games industry. Results show that the behavior of forward looking consumers has a signi cant impact on pricing.

Varian and Shapiro [18] de nes information good as "anything that can be digitized - encoded as a stream of bits". In line with this de nition, here we categorize games delivered as-a-service as information good. Pricing information goods generally involves non-linear price structures [18, 5, 25, 22]. It is also quite common to observe multi part tari s like at rate pricing, two part tari s, etc. in the pricing of information goods [12, 21]. If we consider gaming-on-demand usage to be the number of hours a game is played, then non-linear usage based pricing can be used to price gaming-on-demand services. Software vendors who provide software-as-a-service have adopted this pricing model, for example, Salesforce.com. Firms like Amazon.com which provide computing infrastructure-as-a-service have adopted non-linear pricing which has a combination of usage based and xed fee components. As with the case of other information goods, gaming-as-a-service exhibits zero marginal cost as the cost of providing an additional unit of a game for an unit time is essentially zero. However, cloud gaming providers have to incur a cost to keep track of the usage of the consumer, which is best expressed as transaction cost. Non-linear pricing theory states that the optimal pricing policy of a monopolist should always be based on usage [14, 25]. In a generalized discussion on non-linear pricing of information goods, Sundararajan [21] has shown that if we consider the near zero marginal costs of information goods along with the costs of administering a usage based pricing schedule it is possible to explain the profitability of fixed fee pricing for information goods. In this paper, we make use of the above observation to consider fixed-fee pricing (independent of usage) as well as usage based pricing as possible pricing strategies for gaming-on-demand providers.

In this paper we are interested in determining the optimal pricing structure for gaming-on-demand providers by taking in to account the quality of broadband services available to the gamers. We model heterogeneous gamers characterized by their propensity to engage in gaming, and the quality of their broadband services, in order to develop a pricing schedule. This work contributes to the literature on pricing of cloud gaming services and provides guidelines for cloud gaming providers on pricing their offerings.

In Section 2, we introduce the basic notations and assumptions used in rest of the paper. Pricing plans considered in this paper are elaborated in Section 3. We explain the selection problem of a gamer in Section 4. In Sections 5 to 7, we identify optimal usage based and fixed fee pricing plans for both gamer and cloud game service provider. We take an example in Section 8 to show the applicability of the closed form expressions derived for optimal pricing plans in the previous sections. In Section 9, we conclude after highlighting the application areas and the key contributions of this paper.

2 Model

In this paper, a monopoly cloud gaming provider offers gaming as a service to consumers (gamers). We assume that the variable cost of offering an additional unit of a game as a service for a unit time is zero. We also assume a cost of administering usage based fee incurred by cloud gaming providers, and call it transaction costs. From the perspective of a cloud gaming provider, gamers are characterized using two parameters: gamer type ( ) and broadband non-uniformity( ). Gamer type ( ) indicates propensity of a gamer toward gaming, whereas broadband non-uniformity is defined as the change in data rate of the broadband connection in unit time, and serves as the measure for quality of the broadband service. Mathematically, is represented as $j = t$, the absolute value of the change in the data rate ( ) from time period t to time period $t+\ \ t$. The gamers are heterogeneous, and we index them by their type $2\ \ \_; \ ^{-}$ and broadband non-uniformity $2\ [\_;\ ^{-}]$. The utility function of a gamer with type and broadband non-uniformity is represented by $U(q(\ ;\ );\ ;\ )$. From the functional form of utility function, utility gained by a gamer is dependent on its identi able characteristics, i.e. and , and q, the quantity of games played (consumed) from her cloud gaming provider. As discussed earlier, the quantity consumed is the number of hours a game is played. Corresponding net utility for the gamer is expressed as $U(q(\ ;\ );\ ;\ )$ where is the price paid by the gamer for the quantity of games consumed. In the following discussion, numbered subscripts to functions denote the partial derivatives with respect to the corresponding arguments. For example, $U_1(q(\ ;\ );\ ;\ )$ is the rst order partial derivative of utility function U with respect to the rst argument, q, while $U_{11}(q(\ ;\ );\ ;\ )$ is the second order derivative with respect to q. $U_{12}(q(\ ;\ );\ ;\ )$ represents the cross partial derivative of utility function U with respect to the

.9552 Tf 4.552 rst 2(0) and the [()]TJ/F15 10.406033 ( ) ;)]TJ/F15 11.9552 Tf 23.609 0 Td [()-284(represen827(ts)-284(the)78

(i) $U^L(0; \ ) = U^H(0; \ ) = 0; \ U^L_1(q(\ ); \ ) \quad 0; U^H_1(q(\ ); \ ) \quad 0 \ ; U^L_{11}(q(\ ); \ ) < 0;$
$U^H_{11}(q(\ ); \ ) < 0 \ 8 \ q > 0$

(ii) $U^L(q(\ ); \ ) \quad U^H(q(\ ); \ ) \ 8 \quad 2 \quad \_; \ ^-$

(iii) $U^L_2(q(\ ); \ ) > 0; U^H_2(q(\ ); \ ) > 0; U^L_{12}(q(\ ); \ ) > 0; U^H_{12}(q(\ ); \ ) > 0 \ 8 \ q > 0$

(iv) $\lim_{q \ ! \ 1} U^L(q(\ ); \ ) = V^L(\ ) \ < \ 1 \ ; \lim_{q \ ! \ 1} U^H(q(\ ); \ ) = V^H(\ ) \ < \ 1$

3 Pricing Plans

In this paper, we consider two di erent pricing plans o ered by cloud gaming providers: xed fee and usage based fee.

(i) Fixed fee: A gamer pays a pre-speci ed xed amount T for unlimited consumption of games for a speci c time period.

(ii) Usage based fee: In this plan, there is a price for each unit time period of game played, and the entire schedule of quantity price pairs is available to the gamers. From the revelation

4 Selection Problem of a Gamer

In this section we determine the conditions under which gamers adopt a xed fee plan in the presence of an incentive compatible usage based plan. We rst establish some initial results related to usage based plan. Unless otherwise stated, all proofs are presented in the appendix.

Lemma 1. *If $q(\; ; \;)$ denotes the consumption of a gamer (characterized by gamer type and broadband broadband disruption) who has opted for an incentive compatible plan, then:*

(a) $q_1(\; ; \;) \quad 0$.

(b) $q_2(\; ; \;) \quad 0$.

Lemma 2. *If preference function of a gamer is de ned as*

$F(q(\; ; \;); \; ; \;) = U(q(\; ; \;); \; ; \;) \quad (\; ; \;)$*, then:*

(a) $F(q(\; ; \;); \; ; \;)$ *is strictly increasing in* .

(b) $F(q(\; ; \;); \; ; \;)$ *is non-increasing in* .

$(q(\; ; \;)) =$

For limiting cases of broadband non-uniformity, preference functions are de ned as $F^L(q(\;); \;)$ (for $\; ! \; 0$) and as $F^H(q(\;); \;)$ (for $\; ! \; 1$). Accordingly Lemma 2 is modi ed as follows:

Lemma 3. *For limiting cases of user variability, if preference functions are de ned as* $F^L(q(\;); \;) = U^L(q(\;); \;) \quad (\;)$ *and* F^L)

Here, we assume that the gamer who is indi erent will opt for the xed fee plan. The left hand side of Equation 2 is de ned as *Fixed Fee Surplus*. If the xed fee surplus is more than or equal to the xed fee T, then the gamer opts for xed fee plan instead of usage based plan.

Lemma 4. *If the xed fee surplus is de ned as* $X(q(\; ; \;); \; ; \;) = V(\; ; \;) \quad U(q(\; ; \;); \; ; \;) + (\; ; \;)$*, then the following results are established.*

(a) $X(q(\; ; \;); \; ; \;)$ *is strictly increasing in*

(b) $X(q(\; ; \;); \; ; \;)$ *is strictly decreasing in*

Lemmas 3.871 Td [(Lemma)-350(in)]TJ/73

4.1.1 Limiting Cases

In this section we look at the decision problem of a gamer in the limiting cases of broadband non-uniformity.We rst establish the following lemma to de ne the characteristics of xed fee surplus in limiting cases.

Lemma 5. *For limiting cases of broadband non-uniformity, if xed fee surpluses are de ned as*

$X^L(q( );) = V^L( )\quad U^L(q( );) + ( )$ *and*

$X^H(q( );) = V^H( )\quad U^H(q( );) + ( )$, *then*

(a) $X^L(q( );)$ *is strictly increasing in*

(b) $X^H(q( );)$ *is strictly increasing in*

Lemmas 3 and 5 lead to Proposition 2. We use the following de nitions of gamer types for limiting cases in Proposition 2.

Gamer types ^L_U and ^H_U are de ned as:

$q( ) = 0\ 8 < {}^L_U$ when $! 0$ and $q( ) = 0\ 8 < {}^H_U$ when $! 1$.

Types ^L_S and ^H_S are de ned as:

$^L_S = Minf : V^L( )\quad U^L(q( );) + ( ) = Tg$ and

$^H_S = Minf : V^H( )\quad U^H(q( );) + ( ) = Tg$

Proposition 2. *Given an option to choose between two plans: usage based and xed fee, a gamer's choice will follow the conditions given below:*

(a) If $V^L(\quad {}^L$H(

If $V^H(\_) \quad T < U^H(q(\_);\_) \quad (\_)$ and $V^H(\bar{\ }) \quad T \quad U^H(q(\bar{\ });\bar{\ }) \quad (\bar{\ })$, then gamers of type $[\ ^H_U$

gamer who has access to a steady broadband connection will opt for a xed fee plan even if her gamer type is relatively low.

Propositions 2(d) and 2(e) taken together imply that for a high level of broadband non-uniformity we can expect both adoption of cloud gaming and the shift from usage based plan to xed fee plan to occur at higher values of gamer type. Gamers with a relatively better quality broadband connection (low broadband non-uniformity) will not only adopt cloud gaming at lower values of gamer type but also shift to the xed fee plan from the usage based plan earlier.

5 Optimal Usage Based Pricing Plan by Cloud Gaming Providers

In this section we determine the pricing plan o ered by the cloud gaming providers which maximizes its pro ts. We rst look at the scenario where the gaming provider o ers a usage based plan only. We also assume that the transaction cost is in the form $c(q(\;;\;))$. In the absence of any xed fee plan, if $q(\;;\;)$ is the optimal quantity of games consumed and $(q(\;;\;))$ is the price charged by the gaming provider, then the following proposition determines the optimal price-quantity combination.

Proposition 3. *We de ne $_U$ and $_H$ as follows:*

$$q(\;;\;) = 0 \quad 8 \quad < {}_U \tag{3}$$

$$q(\;;\;) = 0 \quad 8 \quad > {}_H \tag{4}$$

$_U$ is the value of gamer type below which the quantity of games consumed, q is zero, irrespective of the value of . Similarly, $_H$ is the value of broadband non-uniformity above which the quantity of games consumed q is zero, irrespective of the value of . Using this de nition of $_U$ and $_H$, optimal quantity of game consumed by a gamer of type and having broadband

non-uniformity is calculated by solving the following unconstrained optimization problem:

$$
\begin{aligned}
\max_{q(:;:;:)} \quad & \int_{U} h(\ ) \int^{H} [U(q(\ ;\ );\ ;\ ) \quad c(q(\ ;\ ))]g(\ )d\ d\ + \\
& G(\_) \int_{U} h(\ ) \int_{U} \int^{H} [U_{12}(q(x;y);x;y) \!:\! q_1(x;y)]dydxd\ + \\
& G(\_) \int_{U} h(\ ) \int_{U} \int^{H} U_{23}(q(x;y);x;y)dydxd \\
& \int_{U} h(\ ) \int_{\_}^{H} G(\ ) \int_{U} [U_{12}(q(x;\ );x;\ ) \!:\! q_1(x;\ )]dxd\ d \\
& \int_{U} h(\ ) \int_{\_}^{H} G(\ ) \int_{U} U_{23}(q(x;\ );x;\ )dxd\ d
\end{aligned}
$$

Optimal pricing plan for optimal quantity of game consumed (q (;)) is de ned by the following expression:

$$
(q\ (\ ;\ )) = U(q\ (\ ;\ );\ ;\ ) \quad \int_{U} \int^{H} [U_{12}(q(x;y);x;y) \!:\! q_1(x;y) + U_{23}(q(x;y);x;y)]dydx \tag{5}
$$

where $h(\ )$ and $g(\ )$ charaterize the density functions of gamer type and broadband non-uniformity for di erent gamers. Individual rationality condition is satis ed in following ranges of and : 2 [$_U$;] and 2 [_; $_H$].

A cloud gaming provider will o er cloud games at a price which is greater than or at least equal to the transaction cost that the it incurs in o ering that service. Therefore, if the transaction cost is high for the cloud gaming provider, then the price charged is higher too which results in a decrease in the number of adopters.

6 Utility of Fixed Fee Plan for Cloud Gaming providers

In Section 4.1, we established some results to prove the willingness of gamers to opt for a xed fee plan under certain conditions. In this section, we see the incentive of the cloud gaming provider in o ering a xed fee plan. In the absence of xed fee, we de ne the optimal quantity of games consumed by a gamer with and as q (;). In usage based plan, a cloud gaming provider incurs a transaction cost by monitoring quantity of games consumed per unit time and we denote this transaction cost as $c(q\ (\ ;\ ))$. In this paper, we assume zero transaction cost for xed fee plan. Based on this set of conditions, we establish the following proposition.

Proposition 4. *If the transaction cost of monitoring the usage of games is non-zero, then it* *is always pro t improving for the cloud gaming provider to o er xed fee plan along with usage based plan.*

Proof. In absence of any xed fee plan q (;) is the optimal quantity of games consumed by a gamer (characterized by and) and corresponding cost incurred by the gamer is (;).

Proposition 5. *For a gamer with broadband non-uniformity* $_M$*, if we assume that the gamer shifts to xed fee plan from the usage based plan at a gamer type* $_M$*, where* $_M \; 2 \; [\, ^L_S; \; ^H_S]$*, then the optimal combination of usage based fee and xed fee can be determined as follows.*

(a) $U_1(q \; (\; ; \; _M)$

The value of optimal usage q is determined by using Proposition 3. The unconstrained optimization problem for calculating the optimal quantity can be expressed in the form $\int_U \int_{\_}^H I \, dy d$ where integrand I is defined as:

$$I = U(q(\; ; y); \; ; y) g(y) h(\;) \quad c(q(\; ; y)) g(y) h(\;)$$

$$h(\;) \int_U [U_{12}(q(x; y); x; y) q_1(x; y) \quad U_{23}(q(x; y); x; y)][G(y) \quad G(\_)] dx \qquad (6)$$

To determine q we differentiate integrand I with respect to q and equate it to 0 using the first order condition.

$$\frac{dI}{dq} = ((\quad) \quad q \quad c) \quad e^{(\; (\; + \; y))} \quad e \quad e \; - \; e \; {}^{y} \frac{d}{dq} \int_U q_1(x; y) dx$$

Simplifying this equation we get,

$$q = \qquad c \quad \frac{(e \; - \; e \;)}{e}$$

Using the value of q to calculate from Proposition 3,

$$(\; ; \;) = U(q \; (\; ; \;); \; ; \;) \int_U \int^H [U_{12}(q(x; y); x; y) \text{:} q_1(x; y) + U_{23}(q(x; y); x; y)] dy dx$$

For the utility function in the example, $U_{12} = 1$ and $U_{23} = 0$. Therefore, we get

$$(\; ; \;) = U(q \; (\; ; \;); \; ; \;) \int^H q(\; ; y) dy$$

$$(\; ; \;) = U(q \; (\; ; \;); \; ; \;) \quad (\quad c + \; {}^{1}$$

9 Concluding Discussions

This work is an attempt to address the issue of pricing cloud gaming o erings. The pricing decision is modeled by introducing two important factors - gamer type or propensity of a gamer to engage in cloud gaming, and non-uniformity of the gamer's broadband connection - which reduce the transaction costs, they should also adjust the usage based pricing to increase the fraction of adopters.

In future, we plan to analyze the e ect of combined fee plan on the decision of gamers. In some cases of combined fee plans of IaaS providers, the availability of computing resources is guaranteed (Amazon EC2) [http://aws.amazon.com/ec2/purchasing-options/]; for some other cases these contracts are bundled with a specialized consulting service (Rackspace) [http://www.rackspace.com/cloud/cloud_hosting_products/servers/pricing]. These special

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Appendix

1 Selection Problem of a Gamer

Lemma 1. *If $q(\ ;\ )$ is the quantity of games consumed by a gamer who has opted for an incentive compatibility plan, then:*

(a) $q_1(\ ;\ ) > 0$.

(b) $q_2(\ ;\ )\ 6\ 0$.

Proof of part (a). Let us assume $q_1(\ ;\ ) < 0$. Therefore $q(\ ;\ ) > q(\ \ +\ ;\ )$ for $\ > 0$. As the plan is incentive compatible, from condition [IC]

$$U(q(\ ;\ );\ ;\ )\qquad(\ ;\ ) > U(q(\ \ +\ ;\ );\ ;\ )\qquad(\ \ +\ ;\ ) \tag{1}$$

From condition [IC] for a gamer with type $\ +\ $ and broadband non-uniformity ,

$$U(q(\ \ +\ ;\ );\ \ +\ ;\ )\qquad(\ \ +\ ;\ ) > U(q(\ ;\ );\ \ +\ ;\ )\qquad(\ ;\ ) \tag{2}$$

Adding up Inequalities 1 and 2 yields the following inequality:

$$U(q(\ \ +\ ;\ );\ \ +\ ;\ )\quad U(q(\ ;\ );\ \ +\ ;\ ) > U(q(\ \ +\ ;\ );\ ;\ )\quad U(q(\ ;\ );\ ;\ ) \tag{3}$$

Inequation 3 implies that $U_2(q(\ ;\ );\ ;\ )\ 6\ 0$ as $q(\ ;\ ) > q(\ \ +\ ;\ )$ - a contradiction of utility function property, which completes the proof. □

Proof of part (b). Let us assume $q_2(\ ;\ ) > 0$, Therefore $q(\ ;\ \ +\ ) > q(\ ;\ )$ for $\ > 0$. Using condition [IC],

$$U(q(\ ;\ );\ ;\ )\qquad(\ ;\ ) > U(q(\ ;\ \ +\ );\ ;\ )\qquad(\ ;\ \ +\ ) \tag{4}$$

Following condition [IC] for a gamer with type and broadband non-uniformity $\ +\ $,

$$U(q(\ ;\ \ +\ );\ ;\ \ +\ )\qquad(\ ;\ \ +\ ) > U(q(\ ;\ );\ ;\ \ +\ )\qquad(\ ;\ ) \tag{5}$$

Adding up Inequalities 4 and 5 yields the following inequation:

$$U(q(\ ;\ \ +\ );\ ;\ \ +\ )\quad U(q(\ ;\ );\ ;\ \ +\ ) > U(q(\ ;\ \ +\ );\ ;\ )\quad U(q(\ ;\ );\ ;\ ) \tag{6}$$

Inequation 6 implies that $U_3(q(\ ;\ );\ ;\ ) > 0$ as $q(\ ;\ \ +\ ) > q(\ ;\ )$ - a contradiction of utility function property, which completes the proof. □

1.1 Selection of usage based pricing Plan

Lemma 2. *If preference function of a gamer is de ned as*
$F(q(\ ;\ );\ ;\ ) = U(q(\ ;\ );\ ;\ )\qquad(\ ;\ )$, *then:*

(a) $F(q(\ ;\ );\ ;\ )$ *is strictly increasing in .*

(b) $F(q(\ ;\ );\ ;\ )$ *is non-increasing in .*

Proof. Applying rst order condition to satisfy condition [IC],

$$U_1(q(\; ; \;); \; ; \;).q_1(\; ; \;) \quad {}_1(\; ; \;) = 0 \tag{7}$$

$$U_1(q(\; ; \;); \; ; \;).q_2(\; ; \;) \quad {}_2(\; ; \;) = 0 \tag{8}$$

Di erentiating $F(q(\; ; \;); \; ; \;)$ with respect to yields:

$$F_2(q(\; ; \;); \; ; \;) = U_1(q(\; ; \;); \; ; \;).q_1(\; ; \;) + U_2(q(\; ; \;); \; ; \;) \quad {}_1(\; ; \;) \tag{9}$$

Using the results obtained in Equation 7, Equation 9 results into:

$$F_2(q(\; ; \;); \; ; \;) = U_2(q(\; ; \;); \; ; \;) \tag{10}$$

As $U_2(q(\; ; \;); \; ; \;) > 0$, $F_2(q(\; ; \;); \; ; \;) > 0$

To prove part (b), di erentiating $F(q(\; ; \;); \; ; \;)$ with respect to yields:

$$F_3(q(\; ; \;); \; ; \;) = U_1(q(\; ; \;); \; ; \;).q_2(\; ; \;) + U_3(q(\; ; \;); \; ; \;) \quad {}_2(\; ; \;) \tag{11}$$

Using the results obtained in Equation 8, Equation 10 results into:

$$F_3(q(\; ; \;); \; ; \;) = U_3(q(\; ; \;); \; ; \;) \tag{12}$$

As $U_3(q(\; ; \;); \; ; \;) \leqslant 0$, it shows that $F_3(q(\; ; \;); \; ; \;) \leqslant 0$

□

Lemma 3. *For limiting cases of user variability, if preference functions are de ned as* $F^L(q(\;); \;) = U^L(q(\;); \;) \quad (\;)$ *and* $F^H(q(\;); \;) = U^H(q(\;); \;) \quad (\;)$*, then*

(a) $F^L(q(\;); \;)$ *is strictly increasing in .*

(b) $F^H(q(\;); \;)$ *is strictly increasing in .*

Proof. Applying rst order condition to satisfy condition [IC],

$$U^L_1(q(\;); \;).q_1(\;) \quad {}_1(\;) = 0 \tag{13}$$

Di erentiating $F^L(q(\;); \;)$ with respect to yields:

$$F^L_2(q(\;); \;) = U^L_1(q(\;); \;).q_1(\;) + U^L_2(q(\;); \;) \quad {}_1(\;) \tag{14}$$

Using Equation 13, Equation 14 can be rewritten as:

$$F^L_2(q(\;); \;) = U^L_2(q(\;); \;) \tag{15}$$

As $U^L_2(q(\;); \;) > 0$, it shows that $F^L_2(q(\;); \;) > 0$ □

Part (b) can be established analogously.

Proof of Part (a). Di erentiating $X(q(;); ;)$ w.r.t

$$X_2(q(;); ;) = V_1(;) \quad U_1(q(;); ;):q_1(;) \quad U_2(q(;); ;) + {}_1(;) \tag{16}$$

From Equation 7, Equation 16 simpli es to:

$$X_2(q(;); ;) = V_1(;) \quad U_2(q(;); ;) \tag{17}$$

$$X_2(q(;); ;) = \lim_{q!1} U_2(q(;); ;) \quad U_2(q(;); ;) \tag{18}$$

As $U_{12}(q(;); ;) > 0$, it proves that $X_2(q(;); ;) > 0$ □

Proof of Part (b). Di erentiating $X(q(;); ;)$ w.r.t

$$X_3(q(;); ;) = V_2(;) \quad U_1(q(;); ;):q_2(;) \quad U_3(q(;); ;) + {}_2(;) \tag{19}$$

From Equation 8, Equation 19 simpli es to:

$$X_3(q(;); ;) = V_2(;) \quad U_3(q(;); ;) \tag{20}$$

$$X_3(q(;); ;) = \lim_{q!1} U_3(q(;); ;) \quad U_3(q(;); ;) \tag{21}$$

As $U_{13}(q(;); ;) < 0$, $X_3(q(;); ;) < 0$ □

1.2.1 Limiting Cases

Lemma 5. *For limiting cases of broadband non-uniformity, if xed fee surpluses are de ned as*
$X^L(q( );) = V^L( ) \quad U^L(q( );) + ( )$ *and*
$X^H(q( );) = V^H( ) \quad U^H(q( );) + ( )$*, then*

(a) $X^L(q( );)$ *is strictly increasing in*

(b) $X^H(q( );)$ *is strictly increasing in*

Proof of Part (a). Di erentiating $X^L(q( );)$ w.r.t

$$X_2^L(q( );) = V_1^L( ) \quad U_1^L(q( );):q_1( ) \quad U_2^L(q( );) + {}_1( ) \tag{22}$$

From Equation 13, Equation 22 simpli es to:

$$X_2^L(q( );) = V_1^L( ) \quad U_2^L(q( );) \tag{23}$$

$$X_2^L(q( );) = \lim_{q!1} U_2^L(q( );) \quad U_2(q( );) \tag{24}$$

As $U_{12}^L(q( );) > 0$, it proves that $X_2^L(q( );) > 0$ □

Part (b) can be established analogously.

Proposition 2. *Given an option to choose between two plans: usage based and xed fee, a gamer's choice will follow the conditions given below:*

(a) If $V^L(\_) \quad T > U^L(q(\_);\_) \quad (\_)$ *or* $V^H(\_) \quad T > U^H(q(\_);\_) \quad (\_)$*, then all gamers opt for xed fee plan.*

(b) If $V^L(^-) \quad T < U^L(q(^-);^-) \quad (^-)$ *or* $V^H(^-) \quad T < U^H(q(^-);^-) \quad (^-)$*, then all gamers opt for usage based plan.*

(c) If $V^L(\_) \quad T < U^L(q(\_);\_) \quad (\_)$ *aiw 0 0 m 5.64 0 l SQBT/F15 10.9091 Tf 414.773 202.086 Td [())]TJ/F4*

(d) ${}^{H}_{U} > {}^{L}_{U}$

(e) ${}^{H}_{S} > {}^{L}_{S}$

Revenue responses ${}^{L}_{U}$ *and* ${}^{H}_{U}$ *are de ned as:*
$q(\ ) = 0\ 8 < {}^{L}_{U}$ *when* $! \ 0$ *and* $q(\ ) = 0\ 8 < {}^{H}_{U}$ *when* $! \ 1$.
Revenue responses ${}^{L}_{S}$ *and* ${}^{H}_{S}$ *are de ned as:*
${}^{L}_{S} = Minf\ : V^{L}(\ )\quad U^{L}(q(\ );\ ) + (\ ) = Tg$ *and*
${}^{H}_{S} = Minf\ : V^{H}(\ )\quad U^{H}(q(\ );\ ) + (\ ) = Tg$

Proof of Part (a). Using results found in Lemma 5, xed fee surplus increases with increasing for limiting conditions of . Hence if a gamer of type $\underline{\ }$ adopts xed fee plan, then all gamers (with any $> \underline{\ }$) will adopt xed fee plan because of higher xed fee surplus. This concludes the proof for part (a) of Proposition 2. □

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2 Optimal Usage Based Pricing Plan by Cloud Gaming Providers

Proposition 3. *The optimal quantity* $(q$ $($

$$H_{xy} = H_{yx} = U_{11}(q(x;y); \; ; \;)q_1(x;y)q_2(x;y) + U_1(q(x;y); \; ; \;)q_{12}(x;y) \qquad _{12}(x;y) \tag{42}$$

Substituting and as solutions in Equation 40,

$$U_{11}(q(\; ; \;); \; ; \;)(q_1(\; ; \;))^2 + U_1(q(\; ; \;); \; ; \;)q_{11}(\; ; \;) \qquad _{11}(\; ; \;) < 0 \tag{43}$$

Di erentiating Equation 38 with respect to and substituting the er5 10.9091 Tf4H yx = U U U) 4.))2 + U1(U ; ;)(1(

From Equation 48,

$$(\; ; \;) = U(q(\; ; \;); \; ; \;) \int_{\cup} \int^{H} [U_{12}(q(x;y);x;y)q_1(x;y) + U_{23}(q(x;y);x;y)]dydx \tag{56}$$

Objective function of cloud gaming provider:
Objective function of cloud gaming provider to maximize pro t,

$$\max_{q(:;:);\ (:;:)} \int_{\cup} \int_{\_}^{H} [\; (\; ; \;) \quad c(q(\; ; \;))]f(\; ; \;)d \; d \tag{57}$$

Substituting the expression of $(\; ; \;)$ from Equation 56,

$$\max_{q(:;:)} \int_{\cup} \int_{\_}^{H} [U(q(\; ; \;); \; ; \;) \int_{\cup} \int^{H} [U_{12}(q(x;y);x;y)q_1(x;y) + U_{23}(q(x;y);x;y)]dydx$$
$$c(q(\; ; \;))]f(\; ; \;)d \; d \tag{58}$$

Assuming independence of density functions, i.e. $f(\; ; \;) = h(\;)g(\;)$, objective function becomes,

$$\max_{q(:;:)} \int_{\cup} \int_{\_}^{H} [U(q(\; ; \;); \; ; \;) \int_{\cup} \int^{H} [U_{12}(q(x;y);x;y)q_1(x;y) + U_{23}(q(x;y);x;y)]dydx$$
$$c(q(\; ; \;))]h(\;)g(\;)d \; d \tag{59}$$

$$= \max_{q(:;:)} \int_{\cup} \int_{\_}^{H} [U(q(\; ; \;); \; ; \;) \quad c(q(\; ; \;))]h(\;)g(\;)d \; d \quad \int_{\cup} \int_{\_}^{H} E(\; ; \;)h(\;)g(\;)d \; d \tag{60}$$

with $E(\; ; \;)$ is de ned as:

$$E(\; ; \;) = \int_{\cup} \int^{H} [U_{12}(q(x;y);x;y)q_1(x;y) + U_{23}(q(x;y);x;y)]dydx \tag{61}$$

Hence,

$$\frac{dE(\; ; \;)}{d}.d \; = dE(\; ; \;) = \frac{d}{d}[\int_{\cup} \int^{H} [U_{12}(q(x;y);x;y)q_1(x;y) + U_{23}(q(x;y);x;y)]dydx]d$$
$$= [\int_{\cup} [U_{12}(q(x; \;);x; \;)q_1(x; \;)dx \quad \int_{\cup} U_{23}(q(x; \;);x; \;)]dx]d \tag{62}$$

Expanding the second part of the integral de ned in Equation 60 and using integration by parts, $\int udv = uv \quad \int vdu$, with $u = E(\; ; \;)$ and $v = G(\;)$,

$$\int_{\cup} \int_{\_}^{H} E(\; ; \;)h(\;)g(\;)d \; d \; = \int_{\cup} f[E(\; ; \;).G(\;)]_{\_}^{H} \quad \int_{\_}^{H} G(\;)dE(\; ; \;)gh(\;)d$$
$$= \int_{\cup} f[\; E(\; ;\_).G(\_)] \quad \int_{\_}^{H} G(\;)dE(\; ; \;)gh(\;)d \tag{63}$$

Substituting expression of $dE(\; ; \;)$ from Equation 62, Equation 63 can be rewritten as:

$$\int_{\cup} E(\; ;\_).G(\_)h(\;)d$$
$$\int_{\cup} \int_{\_}^{H} G(\;)[\int_{\cup} U_{12}(q(x; \;);x; \;)q_1(x; \;)dx \quad \int_{\cup} U_{23}(q(x; \;);x; \;)dx]d \; h(\;)d(\;) \tag{64}$$

$$= \int_{\cup} E(\; ;\_).G(\_)h(\;)d \; + \int_{\cup} h(\;) \int_{\_}^{H} G(\;) \int_{\cup} U_{12}(q(x; \;);x; \;)q_1(x; \;)dxd \; d$$
$$+ \int_{\cup} h(\;) \int_{\_}^{H} G(\;) \int_{\cup} U_{23}(q(x; \;);x; \;)dxd \; d \tag{65}$$

Inserting the expression of integral into Equation 60, objective function becomes:

$$\max_{q(:;:)} \int_{U} h(\ ) \int^{H} [U(q(\ ;\ );\ ;\ )\quad c(q(\ ;\ ))]g(\ )d\ d\ +$$
$$G(\_) \int h(\ ) \int \int^{H} [U_{12}(q(x;y);x;y).q_1(x;y)]dydxd\ +$$
$$G(\_) \int^{U} h(\ ) \int^{U} \int^{H} U_{23}(q(x;y);x;y)dydxd$$
$$\int h(\ ) \int^{U H} G(\ ) \int^{U} [U_{12}(q(x;\ );x;\ ).q_1(x;\ )]dxd\ d$$
$$\int^{U}_{U} h(\ ) \int^{-H}_{\_} G(\ ) \int^{U}_{U} U_{23}(q(x;\ );x;\ )dxd\ d \qquad (66)$$

3 Optimal Pricing Structure in the Presence of Fixed Fee

Proposition 5. *For a gamer with broadband non-uniformity $_M$, if we assume that the gamer shifts to xed fee plan from the usage based plan at a gamer type $_M$, where $_M \ 2\ [\ ^L_S;\ ^H_S]$, then the optimal combination of usage based fee and xed fee can be determined as follows.*

(a) $U_1(q\ (\ ;\ _M);\ ;\ ) = c_1(q\ (\ ;\ _M)) + \frac{1\ F(\ )}{f(\ )}U_{12}(q\ (\ ;\ _M))$

(b) $^M_S = argmax_{^M_S} \int_{U}^{^M_S} [\ (\ ;\ _M)\quad c(q\ (\ ;\ _M))]f(\ )d\ + [1\quad F(\ ^M_S)]$ $[V(\ ^M_S;\ _M)\quad U(q(\ ^M_S;\ _M);\ ^M_S;\ _M) + \ (\ ^M_S;\ _M)]$

(c) $T = V(\ ^M_S;\ _M)\ U(q\ (\ ^M_S;\ _M);\ ^M_S;\ _M) + \ (\ ^M_S;\ _M)$ *Here $q\ (\ ;\ )$ is the optimal quantity and* $) + (q$

$$= \max_{q(\cdot)} \int_{M_U}^{M_S} [U$$

The constrained optimization problem is represented as Lagrangian problem: $L(q(:); T; \ )$

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