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**A new perspective in supply chain coordination**

**by**

**Abhishek Chakraborty**

Doctoral student, IIM Calcutta, Joka, Diamond Harbour Road, Kolkata 700104, India

**&**

**Ashis K. Chatterjee**

Professor, IIM Calcutta, Joka, Diamond Harbour Road, Kolkata 700104, India

## **A new perspective in supply chain coordination**

### **Abstract**

Discount models have been used extensively in the past to achieve coordination between a buyer and a vendor in the context of supply chain management. Such models are based on the vendor offering a discount to the buyer so as to entice him to order in higher batch sizes. The solution is achieved at the point where the vendor is better off and the buyer is not worse off. In this paper we suggest the use of 'reverse discount' as another mechanism for coordination between the buyer and the vendor. The proposed model analyzes the coordination achieved by allowing the buyer to offer a higher price to the vendor, to motivate him in order to reduce the batch size. Various scenarios have been analyzed including determining the net savings a buyer can get through such an increase. The model has been extended to incorporate the case of information asymmetry.

### **Keywords**

Inventory; Supply chain management; Stochastic programming; Production; Nonlinear programming

### **1. Introduction**

A supply chain is comprised of legally independent but interconnected firms, each trying to realize its own objective (Stadler, 2005). As the objectives of the firms are often conflicting in nature, independent planning by the firms may lead to supply chain inefficiencies (Cachon, 2003). In this context, the importance of supply chain coordination among the partners has been stressed by many authors in the recent past (Cachon, 2003 and Dudek, 2004). Central planning, quantity discounts, supply chain contracts (like revenue sharing, buy-back) and credit options are some of the most common mechanisms of achieving coordination in a supply chain (Cachon, 2003, Sarmah et al., 2006). These approaches of achieving coordination have been broadly classified (Albrecht, 2010) as 'strong' and 'weak' form of coordination respectively. Strong form of coordination as is discussed in Cachon (2003), is the set of actions that brings in overall supply chain optimality. Examples include Joint Economic Lot Size models (Goyal, 1977, 1988

and Banerjee, 1986), revenue sharing contracts (Cachon and Lariviere, 2005), and buy back contracts (Pasternack, 1985). Weak form of coordination (Corbett and de Groote, 2000), on the other hand, does not strive for optimality. It is a set of actions that attempts to achieve overall supply chain improvement compared to the solution that would result without these actions. Mechanisms of coordination in weak form include quantity discount models, credit options etc. Strong form of coordination is difficult to achieve in the presence of information asymmetry. This is perhaps the reason that one finds relatively more studies and applications of weak form of coordination (Albrecht, 2010). In this paper an attempt has been made to develop a new mechanism of achieving weak form of coordination.

One of the earliest attempts in achieving supply chain coordination is through quantity discounts. The traditional discounting models (Buffa and Miller, 1979) focus on buyer's response for minimizing his costs subject to the discount schedules offered by the supplier. Other discounting models such as Monahan (1984), Banerjee (1986) and Lee and Rosenblatt (1986) present the vendor's perspective to determine the vendor's quantity discount pricing schedule that will maximize his resulting economic gains without adding any further costs to the buyer. In both the approaches the buyer decides on his economic order quantity and the supplier offers a discount setting up the coordination process. Contrary to these approaches, we develop an approach where the supplier decides on lot sizes and the buyer initiates the process by offering an increase in the wholesale price to motivate the seller to deliver in smaller batch sizes. The vendors deciding on the lot sizes is quite common in many large industries like aerospace industry, automotive industry etc. where the cost of setting up for production is very high (Esmaeilli et al., 2009). As such the coordination mechanism developed may be useful whenever there is a big mismatch between the independent economic lot sizes of the vendor and the buyer. This process may be regarded as a "reverse discount" procedure. We examine the feasibility of coordination that will maximize the buyer's resulting economic gains without altering the costs of the vendor. The model has been extended to incorporate the case of information asymmetry.

The remainder of the paper is organized as follows: A brief review of the relevant literature is

## 2. Literature Review

In a typical buyer-vendor scenario, the buyer would like to operate at his Economic Order Quantity (EOQ) based on his own inventory carrying cost and ordering cost trade-off. The vendor on the other hand, will find this order quantity to be very low (Monahan, 1984). The development of the Joint Economic Lot Size (JELS) models by Goyal (1977, 1988) and Banerjee (1986) is an attempt in supply chain coordination that aims at achieving overall supply chain optimality. These models indicate that overall gains may accrue by considering the 'supply chain point of view' rather than taking the 'buyer's perspective' alone. Detailed reviews of such models can be found in Goyal and Gupta (1989) and Sarmah et al. (2006). However, the JELS solution is not always in the best interests for both the partners as the overall system's improvement is often accompanied by differential benefits to the different partners of the supply chain (Lu, 1995; Sucky, 2005, 2006; Darwish & Odah, 2010). Thus, one of the partners will always be reluctant to go for the JELS models unless being assured of some form of compensation (Goyal and Gupta, 1989). The discounting models assume importance in this context as these models can assure the compensation for the other partner.

The traditional discount models focus only on buyer's perspective. Crowther (1967), Monahan (1984) and Lal and Staelin (1984) were the first to consider the vendor's perspective by developing a model where the vendor offers discounts to the buyer to entice him to increase the batch size. Monahan's model considers a lot-for-lot policy (vendor does not carry any inventory) in which the vendor tries to maximize his resulting gains and compensates the buyer through discounting for the extra inventory holding charges that the buyer incurs. Lee and Rosenblatt (1986) extended the Monahan's model by relaxing the lot-for-lot assumption and hence allowing the vendor to carry inventory as well. Banerjee (1986) further extended the Monahan's model by incorporating a constant production rate for the vendor. Detailed reviews of such type of work can be found in Weng (1995) and Sarmah et al. (2006). Discounting models such as discussed by Li and Liu (2006) or by Shin and Benton (2007) have taken the final demand to be dependent on price and discuss a different philosophy of discounting which is different our focus.

The current paper examines the case of a dominant supplier whose optimal strategy is to set up only once for the production process in the planning horizon owing to his high set up cost. In

such a case it is the buyer who has to carry the entire inventory. The buyer in turn offers the vendor a proposal of increasing the wholesale price as a form of compensation, so as to entice him to reduce the batch sizes and thus, can reduce his high own inventory holding costs. The case of dominant supplier with high cost of setting up of production where it is the vendor who decides on the lot sizes is quite common in many large industries like aerospace industry, automotive industry (Esmaeilli et al., 2009); as such 'reverse discount' mechanism of offering higher wholesale price by the buyer may be useful.

### **3. Mathematical Model**

With the traditional assumption on the production cost structure of the vendor, it is understood that there is a fixed cost of 'set up' every time production is undertaken and there is a variable cost per unit produced. Further, with the assumption of infinite capacity of production, it is apparent that the optimal policy of the vendor facing a uniform deterministic demand will be to 'set up' only once in the entire planning horizon and also not to keep any inventory (note that the inventory is carried by the buyer rather than the vendor). For the buyer on the other hand the tradeoff between ordering costs and inventory costs may be such that it is often optimal to have more than one set up.

The proposed model assumes that the buyer can ask the vendor to increase the number of 'set ups' by offering him an increase in the wholesale price. Thus, he faces a tradeoff between an increase in the wholesale price and ordering costs on the one hand and a reduction in his inventory holding costs on the other hand. The vendor is being benefitted from the additional per unit wholesale price from the buyer but has to incur extra costs through the increased number of 'set ups'. The proposal from the buyer's side will only be accepted by the vendor if he is not worse off as compared to his original optimal plan. If the proposal from the buyer puts the vendor in a worse off position, he will reject such a proposal.

Consider a typical buyer-vendor scenario with the buyer facing a uniform deterministic demand. The costs for the vendor is the unit variable cost and set up cost per production run while for the buyer it is the unit variable cost of purchase, unit holding cost and ordering cost per order placed.

### **3.1. Notations:**

- w      The wholesale price charged initially by the vendor (i.e. unit cost of purchase for the buyer)
- c      The unit variable cost incurred by the vendor to produce his goods
- H      The holding cost in \$ per unit \$ per unit time

Substituting  $D = nQ$  in the above expression we get

$$I = \left( 1 - \frac{wHQ}{2} \right) \frac{H}{2} \quad 0$$

When the buyer chooses  $x = C_s / D$ , his net savings becomes  $\frac{DwH}{4} - C_o - \frac{C_s}{2} - 2 \frac{H}{2}$  ..... (12)

The optimal value for 'x' that will maximize the buyer's cost savings subject to the condition that the vendor is not worse off can be found by solving the following optimization problem:

$$\left. \begin{aligned} \text{Max } & \frac{wHD}{2} - \frac{1}{n} (n-1)C_o - \frac{xHD}{2n} - xD \\ \text{S.T. } & (n-1)C_s - xD = 0 \end{aligned} \right\} \text{..... (13)}$$

### 3.2. Solution Procedure

In the above problem 'n' is an integer and hence techniques for solving non linear programming problems such as Karush-Kuhn Tucker conditions cannot be applied. Now suppose 'n' is a constant, then the problem

$$\begin{aligned} \text{Max } & \frac{wHD}{2} - \frac{1}{n} (n-1)C_o - \frac{xHD}{2n} - xD \\ \text{S.T. } & x = \frac{(n-1)C_s}{D} \end{aligned}$$

can be expressed as

$$\text{Max } \frac{wHD}{2} - \frac{1}{n} (n-1)C_o - \frac{(n-1)C_s}{2n} - \frac{H}{2n} \text{..... (14)}$$

since the expression  $\frac{wHD}{2} - \frac{1}{n} (n-1)C_o - \frac{(n-1)C_s}{2n} - \frac{H}{2n}$  will attain its maximum value at the minimum value for 'x' given by  $(n-1)C_s/D$ .

Computing the value of the above expression for various values of 'n' we can find the point of maximum for 'n' and 'x' respectively (since the expression (14) is concave in 'n').

**Example:** Let us take  $w=25$ ,  $H=0.05$ ,  $D=5000$ ,  $C_o=50$  and  $C_s=500$ . The vendor, when decides to set up only once in the planning horizon, the buyer has to bear the entire inventory. The initial





the buyer will have to be contented with the initial 'threat ve to 91d be contpoint' solution ofe6ted wit44

This is a non linear stochastic programming problem.

Let 'y' be the value of  $C_s$  that will give the buyer maximum expected gain.

The expected gain is given by

was found that such a proposal of price increase from the buyer's side can reduce his costs by 6.95% on an average and which can even go up to 18% under some combinations of parameters.

positive outcomes for both the partners. However, under information asymmetry, there is a trade off of between rejection of the proposal from the vendor's side and an opportunity loss from the buyer's side.

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## APPENDIX

Alternative Solution Procedure by using KKT conditions

Assuming 'n' to be continuous, KKT conditions can be applied to the expression in (13). We will first construct the Lagrangian function as:

$$L(x, n, \lambda) = \frac{wHD}{2} \left( 1 + \frac{1}{n} \right) C_o(n-1) xD + \frac{xHD}{2n} - (n-1)C_s xD \quad \dots\dots\dots (18)$$

Derivation of the KKT conditions (Bazaraa et al., 1993)

$$\frac{\partial L}{\partial x} = D \left( \frac{HD}{2n} + D \right) - (n-1)C_s = 0 \quad \Rightarrow \quad 1 + \frac{H}{2n}$$

$$\frac{\partial L}{\partial n} = (n-1)C_s xD = 0 \quad \Rightarrow \quad x = \frac{(n-1)C_s}{D}$$

$$\frac{\partial L}{\partial n} = \frac{(w-x)HD}{2} \frac{1}{n^2} C_o - C_s = 0 \quad \Rightarrow \quad \frac{C_o}{C_s} = \frac{HD}{2n^2} (w-x)$$

Solving the above optimization problem we get,  $x = \frac{(n-1)C_s}{D}$  and

$$\frac{H}{2n} \left( 1 + \frac{C_o}{C_s} \right) = \frac{HD}{2n^2} (w-x)$$

Since  $\frac{wHD}{2} \left( 1 + \frac{1}{n} \right) C_o(n-1) xD + \frac{xHD}{2n}$  is convex in nature, the KKT point will give the optimal solution.