stories of leveraging IaaS for rapid growth is Zynga, the online gaming publisher [Gan (2010)]. Zynga's initial predictions of the success of FarmVille were much less than what they experienced in reality; for the first 26 weeks FarmVille added 1 million net new users per week. Zynga had run out of in-house data center space within a few weeks after the launch of Farmville, and utilized Amazon EC2 to scale up. Today, Zynga's ability to support 10 million active users per day depends primarily on IaaS providers. Social networking sites experience huge surges in users during certain events, for example, natural disasters, political crises, etc. It is difficult for these companies to invest on computing infrastructure to support such spiky demand as the infrastructure would lie idle after the number of users reduces to initial levels. The above examples serve to highlight the importance of IaaS for those online businesses which experience unexpected and massive fluctuations in number of users on a regular basis. These businesses are primarily two sided platforms where one side, more often the consumer side, is subsidized. The revenue side, advertisers, are charged a premium. An increase in the number of users attracts more advertisers since advertisers can now reach a larger audience. This is known as cross side network effects [Parker and Alstyne (2005)]. The advertising fee increases with more advertisers competing for a limited number of slots, and therefore increase in the number of users indirectly results in higher revenues for the online platform providers. Therefore, it is important that such businesses (online platform providers) have enough computing resources to support a sudden counting for the near zero marginal costs of information goods along with the costs of administering a usage based pricing schedule can explain the profitability of fixed fee pricing. In this paper we model the fluctuations in the number of users of an online platform provider and its consequent impact on the revenues in order to develop a deeper understanding of the pricing policies of IaaS providers. This work contributes to the literature on pricing cloud services and provides guidelines for online platform providers on selection of pricing policies offered.

In Section 2, we introduce the basic notations, definitions and functional properties which we use to prove results in subsequent sections. We introduce the pricing policies considered in this paper in Section 3. The selection problem of online platform provider with two available pricing policies: usage based fee and fixed fee, is discussed in Section 4. Section 5 elaborates on the impact of fixed fee contract towards the profit gained by IaaS providers. We show the shift in online platform provider's preferences when combined fee contract is introduced in Section 6. It also discusses the results found

(i) $U(0; \ ; \ ) = 0; \partial U/\partial q > 0; \partial^2 U/\partial^2 q < 0 \ \forall q$

(ii) $\partial U/\partial \ \ \leq 0; \partial U/\partial \ \ > 0$

(iii) $\frac{\partial^2 U}{\ }$

a pure usage based contract, and therefore the usage will be much higher. However, a buyer will not treat this policy as a xed fee, in nite usage plan. In some cases the availability of computing resources is guaranteed (Amazon EC2); for some other cases these contracts come with a specialized consulting service (Rackspace). In Section 6, we will look at how the combined fee contract a ects the choice of platform providers while deciding the pricing contract.

4 Selection problem of online platform provider

In this section we look at the conditions which dictate the choice of the platform provider when he has two options: to subscribe to the xed fee or to the usage based fee contract. We rst state some initial results associated to usage based contract.

Lemma 1. *If* $q(\ )$ *is the capacity booked using incentive compatible contract, then* $@q=@\ > 0$.

Proof. Lets assume $@q=@\ < 0$, Therefore $q(\ ) > q(\ +\ )$ for $\ > 0$. Using condition [IC],

$U(q(\ );\ ;\ )\quad (q(\ ))\quad))$ 64 0 Td [() Td [(If)ir6If)ir-3(c)51(omp)51(atib)1(le)-273(c)51(or

$$\frac{\partial U}{\partial q};\frac{\partial q}{\partial} > 0; \text{Hence } \frac{\partial}{\partial q} > 0 \quad (6)$$

Equation 6 proves the part (a) of Lemma 2. To prove part (b),

$$\frac{\partial F}{\partial} = \frac{\partial U}{\partial q}.\frac{\partial q}{\partial} + \frac{\partial U}{\partial} \quad \frac{\partial}{\partial q}.\frac{\partial q}{\partial} \quad (7)$$

Using Equation 5, Equation 7 can be rewritten as $\frac{\partial F}{\partial} = \frac{\partial U}{\partial}$. As $\frac{\partial U}{\partial} > 0$, it proves part (b) of Lemma 2, i.e. $\frac{\partial F}{\partial} > 0$. To prove part (c) of Lemma 2, di erentiating $F(q( ); ;)$ w.r.t. yields

$$\partial F/\partial = \partial U/\partial \quad 6\ 0 \quad (8)$$

Hence the third part of the lemma is proved. □

When the online platform provider is given an option of xed fee contract along with usage based contract, he will go for xed fee contract if and only if,

$$V(;) \quad T > U(q( ); ;) \quad (q( )) \quad (9)$$

$$V(;) \quad U(q( ); ;) + \quad (q( )) > T \quad (10)$$

The left hand side of Equation 10 is named as Fixed Fee Surplus. If the surplus is more than or equal to the xed fee rent T, then the online platform provider opts for xed fee contract instead of usage based.

To see the behavior of xed fee surplus with the change in exogenous variables, i.e. and , we state the following results:

Lemma 3. *(a)* $X(q( ); ;) = V(;) \quad U(q( ); ;) + \quad (q( ))$ *is strictly increasing for*

(b) $X(q( ); ;)$ *is strictly decreasing for*

Proof of Part (a).

$$\frac{\partial X}{\partial} = \frac{\partial V}{\partial} \quad \frac{\partial U}{\partial q}\frac{\partial q}{\partial} \quad \frac{\partial U}{\partial} + \frac{\partial}{\partial q}\frac{\partial q}{\partial} \quad (11)$$

From Equation 5, Equation 11 simpli es to:

$$\frac{\partial X}{\partial} = \frac{\partial V}{\partial} \quad \frac{\partial U}{\partial} \quad (12)$$

$$\frac{\partial X}{\partial} = (lim_{q( )\ !\ 1}\frac{\partial U}{\partial}) \quad \frac{\partial U}{\partial} \quad (13)$$

As $\frac{\partial^2 U}{\partial q \partial} > 0$ and $\frac{\partial U}{\partial q} > 0$, $(lim_{q( )\ !\ 1}\frac{\partial U}{\partial}) > \frac{\partial U}{\partial}\ 8\, q( ) < 1$
From Equation 13, it proves that $\frac{\partial X}{\partial} > 0$. □

Proof of Part (b).

$$\frac{@X}{@} = \frac{@V}{@} \quad \frac{@U}{@} \tag{14}$$

$$\frac{@X}{@} = (lim_{q(\ ) \ ! \ 1} \frac{@U}{@}) \quad \frac{@U}{@} \tag{15}$$

As $\frac{@^2U}{@q@} < 0$ and $\frac{@U}{@}$ 6 0, it shows $\frac{@V}{@} < \frac{@U}{@} \ 8 \ q(\ ) < \ 1$
Using these conditions in Equation 14 gives $\frac{@X}{@} < 0$ □

We use results found in Lemmas 2 and 3 to present the following proposition:

(a) $V_L(\ ) \quad T > U_L(q(\ ); \ ) \quad (q(\ ))\ 8 \quad > \quad _L$ *and* $V_L(\ ) \quad T < U_L(q(\ ); \ ) \quad (q(\ ))\ 8 \quad < \quad _L$ *where* $_L$ *is de ned as* $min f \ : V_L(\ ) \quad U_L(q(\ ); \ ) + \ (q(\ )) = T g$

(b) $V_H(\ ) \quad T > U_H(q(\ ); \ ) \quad (q(\ ))\ 8 \quad > \quad _H$ *and* $V_H(\ ) \quad T < U_H(q(\ ); \ ) \quad (q(\ ))\ 8 \quad < \quad _H$ *where* $_H$ *is de ned as* $min f \ : V_H(\ ) \quad U_H(q(\ ); \ ) + \ (q(\ )) = T g$

(c) $_H > \ _L$

Proof of parts (a) and (b). For $\lim \ _{! 0}$, online platform providers with revenue response $\ > \ _L$ will go for xed fee contract because of the property described in Lemma 4 [part(a)] and by de nition of $_L$. Similarly for $\lim \ _{! 1}$, online platform providers with revenue response $\ > \ _H$ will go for xed fee contract because of the property described in Lemma 4 [part(b)] and by de nition of $_H$. □

Proof of part (c). Lets assume that $_H < \ _L$. From the property of $_L$,

$$V_L(\ _L) \quad T > U_L(q(\ _L); \ _L) \quad (q(\ _L)) \tag{22}$$

$$V_L(\ _L) \quad U_L(q(\ _L); \ _L) > T \quad (q(\ _L)) \tag{23}$$

As $\frac{@^2 U}{@q@} < 0$, Equation 23 can be expressed as:

$$V_H(\ _L) \quad U_H(q(\ _L); \ _L) < T \quad (q(\ _L)) \tag{24}$$

As $_H < \ _L$,

$$V_H(\ _H) \quad U_H(q(\ _H); \ _H) < T \quad (q(\ _H)) \tag{25}$$

Equation 25 contradicts the basic property of $_H$, hence $_H > \ _L$. □

Proposition 2 states that for very high values of user variability, online platform providers will opt for the xed fee contract at a higher value of revenue response compared to platform providers with low user variability. The result is intuitive and expected as platform providers who have to deal with high user variability will nd the xed fee contract attractive only when the revenue response is signi cantly high. This is because xed fee contract entails a payment independent of usage and can lead to losses if the user demand is spiky in nature. On the contrary, a platform provider with a steady demand can a ord to opt for a xed fee contract for a relatively lower value of revenue response. This is exactly what we nd in the pricing policy of Amazon's EC2 [`http://aws.amazon.com/ec2/purchasing-options/`].

5 Impact of xed fee

In this section we establish that the pro ts of an IaaS provider will increase

$[q^0(\ );\ \ ^0(\ )]$ is:

[ICC]: $U(q^0(\ );\ ;\ )\ \ T\ \ \ ^0(q^0(\ )) > U(q^0(t);\ ;\ )\ \ T\ \ \ ^0(q^0(t))\ 8t\ \ 2\ \ \_;\ \bar{\ }$ (30)

Lemma 5. *(a)* $Y(q^0(\ );q(\ );\ ;\ )$ *is strictly increasing in .*

(b) $Y(q^0(\ );q(\ );\ ;\ )$ *is strictly decreasing in .*

Proof of part (a).

$$\frac{@Y}{@} = \frac{@U^0}{@q^0}:\frac{@q^0}{@} + \frac{@U^0}{@} \quad \frac{@U}{@q}:\frac{@q}{@} \quad \frac{@U}{@} + \frac{@}{@q}:\frac{@q}{@} \quad \frac{@\ ^0}{@q^0}:\frac{@q^0}{@} \tag{31}$$

[D]enoting)]TJ 0 g 0 G -286.968 -22.515 Td [((b))]TJ 0 g 0 G /F60 10.9091 Tf 19.394 0 Td [(Y)]TJ

Proof of Parts (a) and (b).

Proof of part (b).

$$\frac{@Y_H}{@} = \frac{@U_H^0}{@q^0}.\frac{@q^0}{@} + \frac{@U_H^0}{@} \quad \frac{@U_H}{@q}.\frac{@q}{@} \quad \frac{@U_L}{@} + \frac{@}{@q}.\frac{@q}{@} \quad \frac{@ \ ^0}{@q^0}.\frac{@q^0}{@} \qquad (38)$$

[Denoting $U_H(q(\ ); \ ; \ )$ as U_H and $U_H(q^0(\ ); \ ; \ )$ as U_H^0 for notational convenience]

From the rst order condition of **[ICC-H]**,

$$\frac{@U_H^0}{@q^0}.\frac{@q^0}{@} + \frac{@U_H^0}{@} \quad \frac{@ \ ^0}{@q^0}.\frac{@q^0}{@} = 0 \qquad (39)$$

Using Equations 19 and 39, Equation 38 is reduced to:

$$\frac{@Y_H}{@} = \frac{@U_H^0}{@} \quad \frac{@U_H}{@} \qquad (40)$$

As $\frac{@^2U_H}{@q@} > 0$, so $\frac{@U_H^0}{@} > \frac{@U_H}{@}$

So, $\frac{@Y_H}{@} > 0$ (Hence proved) □

Proposition 5. *For limiting conditions of customer variability, i.e.* lim $_{! 0}$ *and* lim $_{! 1}$, *two revenue responses* c]TJ/F72 10.9091 Tf 19689 T]TJ/F8 10.9866 0T930 m 6.546 0 l SQq1

As $q^0({}^c_L) > q({}^c_L)$ and $\frac{@^2U}{@q@}$, Equation 42 can be expressed as:

$$\lim_{!\ 1} U(q^0({}^c_L); {}^c_L) \quad \lim_{!\ 1} U(q({}^c_L); {}^c_L) < T + {}^0(q^0({}^c_L)) \quad (q({}^c_L))$$

As ${}^c_H < {}^c_L$,

$$\lim_{!\ 1} U(q^0({}^c_H); {}^c_H) \quad \lim_{!\ 1} U(q({}^c_H); {}^c_H) < T + {}^0(q^0({}^c_H)) \quad (q({}^c_H)) \tag{43}$$

Equation 43 contradicts the basic property of ${}^c_H$, hence ${}^c_H > {}^c_L$. □

We now compare the threshold revenue responses of platform providers at limiting conditions of user variability for two di erent set of contracts o ered to online platform providers: rst with usage based and xed fee and second with usage based and combined. Lemma 7 shows our ndings.

Lemma 7. *(a)* ${}^c_H > {}_H$

(b) ${}^c_L > {}_L$

Proof of part (a). Lets assume ${}_H > {}^c_H$. From the property of ${}_H$,

$$V_H( ) \quad T < U_H(q( );) \quad (q( ))\ 8 \quad < {}_H \tag{44}$$

So, $= {}^c_H$ will give the following equation,

$$V_H({}^c_H) \quad U_H(q({}^c_H); {}^c_H) + (q({}^c_H)) < T \tag{45}$$

Again from the property of ${}^c_H$,

$$U_H(q^0({}^c_H); {}^c_H) \quad U_H(q({}^c_H); {}^c_H) + (q({}^c_H)) \quad {}^0(q^0({}^c_H)) > T \tag{46}$$

Now, $V_H({}^c_H) = \lim q( )\ !\ 1\ U_H(q( ); {}^c_H)$ and $\frac{@U}{@q} > 0$

Therefore $V_H({}^c_H) > U_H(q^0({}^c_H); {}^c_H)$ and ${}^0(q^0({}^c_H)) > 0$. As inequality in Equation 46 is valid, Equation 45 cannot be true and it is proved by contradiction. □

Proof of part (b). Lets assume ${}_L > {}^c_L$. From the property of ${}_L$,

$$V_L(r) \quad T < U_L(q( );) \quad (q( ))\ 8 \quad < {}_L \tag{47}$$

So, $= {}^c_L$ will give the following equation,

$$V_L({}^c_L) \quad U_L(q({}^c_L); {}^c_L) + (q({}^c_L)) < T \tag{48}$$

Again from the property of ${}^c_L$,

$$U_L(q^0({}^c_L); {}^c_L) \quad U_L(q({}^c_L); {}^c_L) + (q({}^c_L)) \quad {}^0(q^0({}^c_L)) > T \tag{49}$$

Now, $V_L({}^c_H) = \lim q( )\ !\ 1\ U_L(q( ); {}^c_L)$ and $\frac{@U}{@q} > 0$

Therefore $V_L({}^c_L) > U_L(q^0({}^c_L); {}^c_L)$ and ${}^0(q^0({}^c_L)) > 0$

As inequality in Equation 49 is valid, Equation 48 cannot be true and it is proved by contradiction. □

The interesting result is that platform providers will shift to the combined fee contract at higher values of revenue responses for both the limiting cases. As we have already seen that the impact of introduction of a fixed fee is always profit improving for the IaaS provider, the profits will only increase if there is an additional usage based fee. Therefore, offering a combined fee contract has two effects on the profits of an IaaS provider:

(i)

that Rackspace offers. We plan to incorporate these special incentives in our model to gain a deeper understanding of this issue.

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