INDIAN INSTITUTE OF MANAGEMENT CALCUTTA

WORKING PAPER SERIES

WPS No. 670/ March 2011

An improved mathematical programming formulation for multi-attribute choice behavior

by

Soumojit Kumar
Doctoral student, IIM Calcutta, Joka, Diamond Harbour Road, Kolkata 700104

&

Ashis Kumar Chatterjee
Professor, Indian Institute of Management Calcutta, Joka, Kolkata 700104

An improved mathematical programming formulation for multi-attribute choice behavior

Soumojit Kumar

Doctoral Student, Operations Management

Indian Institute of Management Calcutta

Mobile: 9432215758

Email Id: soumojitk08@iimcal.ac.in

Ashis Kumar Chatterjee

Professor, Operations Management

Indian Institute of Management Calcutta

Email Id: ac@iimcal.ac.in

Abstract: *Conjoint Analysis and Mathematical Programming approaches have been used extensively in the past for modelling multi-attribute choice behavior. The Mathematical Programming approaches are more versatile in their ability to capture complex behavior but have been limited to dealing with objective attributes. Conjoint Anlysis, though limited by the additive utility assumption, allows for both subjective and objective attributes. In the present article, we modify the existing mathematical models to account for situations where the decision maker may base her decisions on*

by the consumer, based on his ranking of different product profiles. The major assumption of product utility as an additive function of utilities of all attributes were common in both CA and the later models. This assumption is based on summative rule as a special type of compensatory

The model with the relevant definitions and notations are presented in Section 2. The resulting integer programming formulation is solved by ILOG CPLEX 10.2 using the data provided by Green and Wind (1975). The results are presented in Section 3 followed by a comparison of Mathematical programming solution and the Conjoint analysis results (Green and Wind 1975) in Section 4. In the concluding section an attempt has been made to highlight the efficacy and versatility of the proposed model vis-à-vis the earlier approaches.

FEW RELEVANT DEFINITIONS

Attributes and Attribute levels- Attributes are the value creating entities that make up the whole product. Attribute levels are the various types of a particular attribute which may be differentiated by certain performance measures or by decision maker's preferential tastes. If we talk about camera quality as an attribute to mobile phones, then two, three or four mega pixels camera form the different levels of the mobile camera attribute. Considering color as attribute different colors like red, yellow or blue will form the attribute levels for the attribute color.

Attributes combined together in a particular combination of respective levels define the whole product or alternative. They can be both subjective and objective. Objective attributes can be ordered according to their levels and customer preferences i.e. straight away we can say one attribute level is better to the next level of the same attribute while the others may not be ordered according to their various levels. Considering motor cycles as a product example, we have price, horse power, fuel efficiency (km/litre) etc. are the objective attributes. A careful observation will show that all of these attributes can be ordered in the customers' preference rating. Lower prices compared to higher price will always be better for a rational customer. Similarly, more fuel efficiency, pick-up will be preferred to less of the same attributes. On the other hand looks, color, brand etc will form subjective attributes and their ordering will be contextual in nature depending upon the preference pattern of individual customers. One cannot presume that yellow is always better to red or vice –versa. These type of attributes cannot be ordered according to their levels and customer preference consistently and vary from consumer to consumer. Their relative order of preference for a particular customer comes out as a solution from the proposed model.

Subjective attributes levels can only be categorized but cannot be ranked on the customers' preferential scale. Hence, subjective attribute levels are nominally scaled. Objective attributes can be ordered according to the preferences of the customers but it cannot be assured that the differences between respective attribute levels are equal in preferential scale of the customers. Thus, objective attribute levels are typically nominally scaled.

Alternatives- Alternatives are the different products in the product line which differ in their attribute combination. More specifically, an alternative can be defined as a vector of attributes. Continuing in the same motor-cycle example we can say different models of motor-cycles like Rajdoot, Bajaj Scooty, Bajaj Pulsar, and Karizzma etc form different alternatives in front of the customers to make a buying decision.

Part-Utility of the attribute levels- Every attribute level will be associated to a utility level by a particular customer. These attribute level utilities will contribute to the total alternative/product utility according to the decision making scheme the customer uses.

Total utility of an alternative- It is the total utility of the product to the customer and is a function of the part-utility of the attributes. The function may be simple additive or a complex function of the part-utility of the attribute levels comprising the alternative. The buying decision of one alternative to the other will be governed by these utility values and more is always preferred to less.

REVIEW OF RELEVANT MODELS

Model 1 (Srinivasan and Shocker's method):- The method assumes simple additive rule for determining the total utility of an alternative and minimizes the total inconsistencies or violations under forced choice preference to obtain the attribute weights.

Here, $A = \{1,2,\dots,j,\dots.n\}$ denotes the set of n alternatives on which pairwise preference judgements are to be made. Each of the alternatives is a vector of *m* attributes defined under the set:

$$P = \{1,2,\dots,p,\dots m\}$$

Also,

$Y_j = (Y_{j1}, Y_{j2}, \dots, Y_{jp}, \dots . Y_{jm})$ denotes the j^{th} alternative. Y_{jp} specifies p^{th} attribute for the j^{th} alternative.

Finally, as specifies earlier under the assumption that each of the m attributes are at least intervally scaled another set is defined

$w = \{w_1, w_2, \ldots, w_p, \ldots, w_m\}$ which denotes the set of attribute weights for the *m* attributes and is the set of decision variables in the model. As the model is assuming simple additive rule for total utility determination the overall utility of an alternative is given by the following expression:

$$s_j = \sum_{p \in t} w_p \, y_{jp}$$

Further, another set is defined as

$\Omega = \{(j, k) : s_j > s_k : (j, k) \in n\}$ which denotes the set of pairwise preference judgements such that the alternative *j* is preferred to *k* in a forced-choice pair comparison from the decision maker under scrutiny.

Finally, Srinivasan and Shocker's model takes the form as under:

$$\text{Maximize} \sum_{(j,k) \in \Omega} z_{jk}$$

$$\text{subject to} \quad s_j - s_k + z_{jk} \geq 0 \;\; \forall (j, k) \in \Omega \qquad (1)$$

$$\sum_{(j,k) \in \Omega} (s_j - s_k) = 1 \qquad (2)$$

Constraint (2) is added to preclude the trivial solution $w_p = 0 \;\forall p \in t$

Model 2 (Threshold Model and suggested extension):- Threshold Model forsakes the simple additive rule that was used by Srinivasan and Shocker (1973) to measure the total utility. Here, another complexity is introduced where a customer will make his/her choice from a subset of offered alternatives and the selection of an alternative in the set will be on the condition that it satisfies certain threshold conditions. The customer or the decision maker may not be able to express the actual threshold conditions and the subset of alternatives he/she actually considering which is going on in the sub-conscious mind.

Similar to the above model the input sets are defined as above i.e. *J, P, Y* and contribute nothing to the overall utility of the alternative. For modeling this type of behavior he suggested to define the overall utility value to be:

$$U_i = \sum_{k} w_k x_k y_{ik}, \ \forall i.$$

In this formulation number of binary variables have increased which are now associated with each of the attributes rather than alternatives as in the former case.

Model 3: Conjoint Analysis: The basic conjoint analysis is represented by the following formulae (Malhotra 2004)

$$U(X) = \sum_{i=1}^{m} \sum_{j=1}^{k_i} \alpha_{ij} x_{ij}$$

where,

$U(X) =$ Overall utility of an alternative

$\alpha_{ij} =$ Part-worth utility associated with the *j*th level of the *i*th attribute.

$k_i =$ Number of levels of attribute *i*.

$x_{ij} =$ 1, if the *j*th level of the *i*th attribute is present

0, otherwise.

PROPOSED MODEL

Consider a multi-attribute choice behavior model where a decision maker is faced with a number of alternatives. An alternative is represented by a number of attributes.The levels of different attributes present in the alternatives determines its relative worth to the decision maker. For attribute such as mileage in context of a car, an alternative having a higher level of mileage is always preferred to an alternative of a lower mileage, everything else remaining same .It may be noted that this is true for ordinal scale data. For attributes such as color no universal ordering may be possible as such the levels of such attributes are nominally scaled. In such cases, levels may be numbered arbitrarily, the inferences from the solutions, however, are to be consistent to the numbering.

The decision maker is presented with a number of product alternatives, and asked to rank them in the order of their preference. Each product alternative is represented by a vector of numbers indicating the levels of the different attributes present in the product. Based on the ranking (or sometimes pairwise comparison) the part utilities of each of the attribute levels are worked out.

Let, ὐ 1,2, … , ὒ, … . ὲ denotes the set of n alternatives on which preference judgements are to be made. Now each of the alternatives is described by the m attributes:

ὒ 1,2, … , ὴ, … ά denotes the complete set of attributes which the alternative set is composed of. ἀ 1,2, … … , ὲ denotes the number of levels of the p^{th} attribute.

The utility function essentially should capture the rationale for the choice behavior of the decision maker. A purely additive model without binary variables would imply compensatory model where the decision maker inherently allows a tradeoff amongst the different attributes. Thus, if U_j denotes the total utility of an alternative j and u_{pk} denotes the part utility of the k^{th} level of the p^{th} attribute then U_j is the summation of the all the part-utilities of the all the underlying attribute levels present in alternative *j*. This part-utilities would normally have a positive non-zero value. However, situations may arise where the decision maker decides based on only a subset of the attributes. This can be taken care of by incorporating binary variables for each of the attributes. A zero value of a binary variable in the final solution would imply that the decision maker does not take into account that attribute into consideration at all.

In our model we have put binary variables not only for each attribute but for each attribute level, which will signify whether a particular attribute level contributes any value to the decision maker or not. We gain extra flexibility in capturing the decision maker's choice at the attribute levels by adding binary variables at every attribute level.

Finally, attributes have been divided into two sets, nominal and ordinal, which can be written as:-

ὒ ὓὒ ‹ ὓὒ

where *NP* is the set of subjective attributes whose levels are nominally scaled and theirTf1.2n701 0 T customer but it may not be possible to know how much exactly one level is better than the other in utility scales.

We define all the pairwise set of the alternatives as

Ω ῦ, ῗ : ῀Y ش ῀Y ش : ῦ, ῗ ᴺ ὐ] which denotes set of pairwise preference judgements such that the alternative *j* is preferred to *k* in a forced-choice pair comparison from the customer.

Given U_j and U_k are the utilities corresponding to j th and kth alternatives respectively where *j* is preferred over *k*, $U_j>U_k$ implies $U_j-U_k = W_{jk}-Z_{jk}$ where Z_{jk} , $W_{jk} >0$ and are respectively inconsistency and consistency.

Minimize ∑ ش,ش ᴺ ي ὤشش

subject to ῀Y ش ῀Y ش ὤششش 0 for all *(j,k)* ᴺΩ (2)

Let us suppose ὣصش όصش ῐ صش for all ῗ ᴺ ἀص , ἡ ᴺ ὕ ὓ

Hence, ὣصش ό صش if ῐ صش 1

0, otherwise, for all ῗ ᴺ ἀص , ἡ ᴺ ὕ ὓ

To linearize the above non-linearity, we use the transformation used in Threshold Model (Mustafi and Xavier 1985) which is:-

ὣصش ῐ صش

ὣصش ῐ صش ό صش 1 for all *k* ᴺἀص, ἡ ᴺ ὕ ὓ

ὣصش ό صش (3)

Similarly we define ὣص ,ص ό ص for all *o* ᴺἀ , *t* ᴺ*OP* and linearize it by the same procedure which is:-

ὣص ,ص

ὣص ,ص ό ص 1 for all έ ᴺ ἀ , ὸ ᴺ ὕ ὓ

ὣص ό ص (4)

To avoid the obvious solution of all *u*'s to be zero we add another constraint which is

∑ ش,ش ᴺ ب ̃Yش ̃Yش 1 (5)

For any attribute given a number of levels ordered low to high, the part-utility of the higher level is always assumed to be higher than the part-utility of the lower level. Hence, the following constraint

ὣطص ὣطشد for all *s>m: s,m* ᴺὰظ , ὸ ᴺ ὕ ὒ̃ (6)

To prevent all the binary variables turning out zero, the following constraint is added

∑ش ᴺ ش ἱ شص ὣ̃ص for all ἡ ᴺ ὕ ὒ̃ (7)

∑ص ᴺ ش , صض ὣ̃ض for all *t* ᴺ*OP* (8)

The values of c_p and c_t are chosen ju tdTc-. Tcs0006 T(w)1d46otioner 8(ly.s0006.735 -172 D0 T55584 686.94 Tm7

Attribute levels	**Package Design**	**Brand Names**	**Prices**	**Good Housekeeping seal**	**Money Back Guarantee**
1	A	K2R	$1.19	Yes	Yes
2	B	Glory	1.39	No	No
3	C	Biessell	1.59	-	-

Table 1: Attributes and possible attribute levels for a carpet cleaner as used by Green(1975)

Alternatives	***Package Design***	***Brand Name***	***Price***	***Good Housekeeping Seal***	***Money-back Guaranteee***	***Respondent's Evaluation (rank number)***
1	*A*	*K2R*	*$1.19*	*No*	*No*	*13*
2	*A*	*Glory*	*1.39*	*No*	*Yes*	*11*
3	*A*	*Bissell*	*1.59*	*Yes*	*No*	*17*
4	*B*	*K2R*	*1.39*	*Yes*	*Yes*	*2*
5	*B*	*Glory*	*1.59*	*No*	*No*	*14*
6	*B*	*Bissell*	*1.19*	*No*	*No*	*3*
7	*C*	*K2R*	*1.59*	*No*	*Yes*	*12*
8	*C*	*Glory*	*1.19*	*Yes*	*No*	*7*
9	*C*	*Bissell*	*1.39*	*No*	*No*	*9*

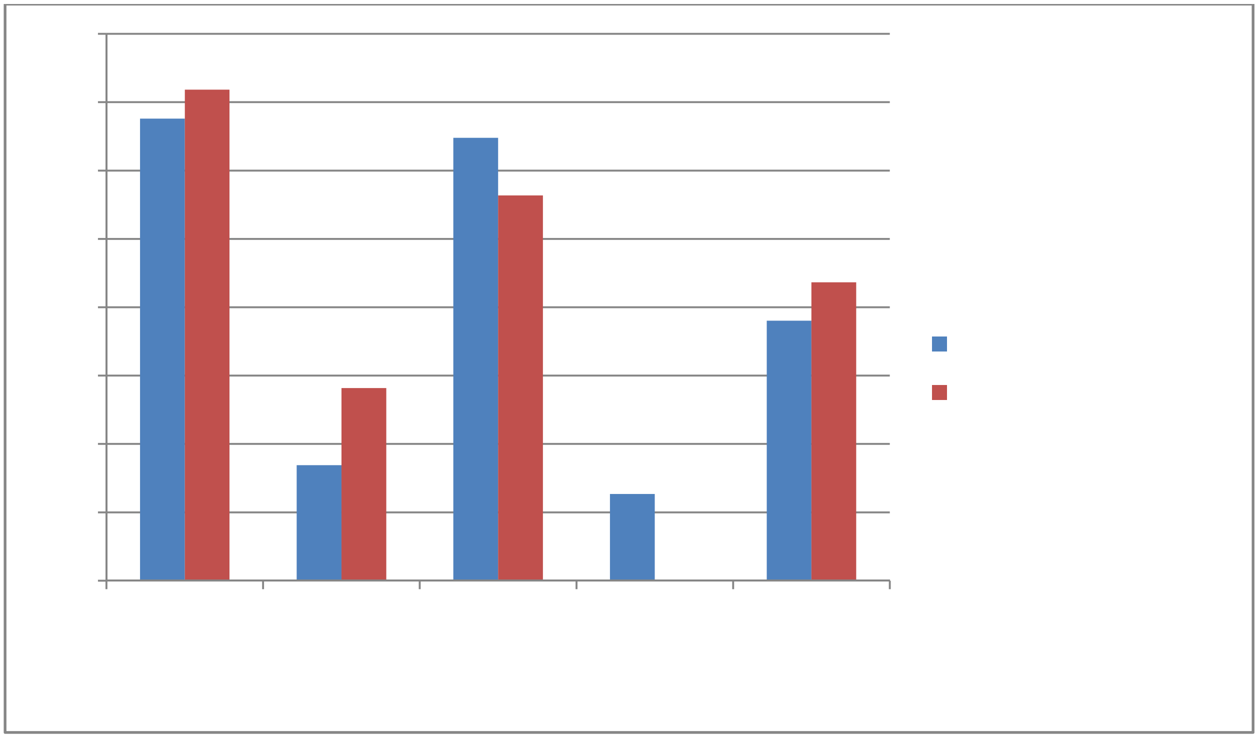
Green and Wind (1975) selected 18 alternatives by using orthogonal array design and they obtained the preference rank of the same from the decision maker. The data on the alternatives together with the decision makers’ prefernce ranking is reproduced in Table 2 above. Conjoint analysis was applied by Green and Wind to find out

	Ỹژ		
	Ỹڈ		
	Ỹٿ		
	Ỹپٹ		
	Ỹپپ		
	Ỹپٮ		
	Ỹپڕ		
	Ỹپٹ		
	Ỹپٿ		
	Ỹپٺ		
	Ỹپژ		
	Ỹپڈ		

Table 4: Total Utilities of the chosen alternatives under experiment.

The problem was also solved using Conjoint Analysis. The final results on relative importance conformed with the results obtained by Green and Wind (1975).

particular attribute, implying that it does not add any value to the decision maker. One may verify that the part-utilities are still positive and the corressponding binary variables have turned out to be zero.(as shown in Table 3, Sl.No. 4). These insights are particularly helpful for a manger because the resources wasted to procure that attributes can easily be shifted to other value adding activities.



Graph 1 : Comparison on the relative importances of the attributes by Conjoint and Proposed Model.

Apart from determining the preference pattern of the decision maker, the other major objective is to utilize the part utilities corresponding to each level of different attributes for deciding on optimal product line. The results for continous part-utilities for both the methods are given in Table 3. The ordering of the levels based on their corresponding part-utility values for any attribute remains the same for that attribute for both the methods. For example, price has three levels as: (L1)$1,19, (L2)$1.39, (L3)$1.59. Ordering the levels based on their part utility values (0.007284, 0.003642, 0) obtained through our proposed model we have L1, L2 and L3 as descending order of preference.The ordering done based on corresponding values obtained on

Conjoint (3.5, 0.666667,-4.16667) yields the same ordering. This goes to reinforce the logical consistency of the two approaches.

Finally from the results of the proposed model on “Package Design” shows clearly that the