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Analytical Aspects of Short-run Growth

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1. Introduction

This paper studies short-run and medium-run aspects of economic growth. The set of comprehensive equilibrium conditions of the long-run is not invoked. Instead of full equilibrium, we deal with contexts that have several Keynesian features such as demand determined output, excess capacity, unemployment and price rigidities. Moreover we shall analyze the 'transition dynamics' associated with out-of-steady-state behaviour.

Much of growth theory is about the long-run (for e.g. Solow [13]). Understanding contemporary growth experience however requires analyzing the short-run. Some full-equilibrium conditions would have to be given up. To take an example, if our interest is in studying recent Indian growth then it seems pointless to assume that there is full capacity utilization and full employment. We need a short-run framework.

Moreover, when analyzing short-run growth we can ill-afford to assume that the rate of growth is constant – or piecewise constant. The literature on Indian growth rates, however, mostly deals with output trends that are log-linear or piece-wise log-linear. That is like assuming that India is always on a steady-state, allowing only for the steady state to shift periodically ("structural breaks") but assuming that the new steady state is attained instantly. A moment's reflection should convince us that this is not the right way to proceed if the rate of growth is varying over time endogenously and the economy is far from being on a steady state. Thinking of an average rate that is constant or piecewise constant is going to be treacherous. We will forever be hunting for 'structural breaks' even though no such thing may have occurred.

The analysis attempted here is theoretical and preliminary. It is best viewed as "experiments" using a set of alternative models. The models are simple and conclusions can be stated in particularly transparent terms. It would be easy to extend these models to more realistic scenarios. The first model is a pure demand-driven model that dynamizes the simple multiplier model of the Keynesian cross.

2. Pure Demand Growth with Excess Capacity and Unemployment

We consider the simple Keynesian multiplier model in a *closed* economy in which there exists excess capacity and unemployment, i.e. the economy is demand-constrained. We assume that investment is wholly autonomous, consumption has an autonomous component and the marginal propensity to consume is constant.

If the consumption function is $C = \overline{C} + cY$, then the equilibrium condition that demand = income gives

$$D = Y = A/s \qquad (1)$$

where $A \equiv I + \overline{C}$

Let $\mu \equiv \frac{I}{I + \overline{C}}$ be the share of investment expenditure in total autonomous demand. For positive I and $\overline{C}$, we have $0 < \mu < 1$. Writing g_x for the rate of growth of x, the relation $A = I + \overline{C}$ implies

0 and let . Then

(1) , (2)

Note that is a function of time such that

— (1) ( ) (1) , where (3)

Fig 2: The growth curve in the demand model

If we were to draw a curve to depict the function $\mu(t)$ when $\beta > \alpha$, or the function $(1- \mu(t))$ when $\alpha > \beta$, that curve will likewise be S-shaped and have an asymptote 1. Such a curve will then display how the shares of investment and consumption in output will behave over time, since $(I/Y) = s\mu$ and $(C/Y) = 1- s\mu$. It is clear that the share of the faster growing demand component will converge to s or $(1 – s)$ as the case may be.

Property II(b): The growth curves of g, μ, (I/Y) and (C/Y) are S-shaped. When $\alpha > \beta$, then μ and $(I/Y$

$$\frac{\theta^2}{4}$$

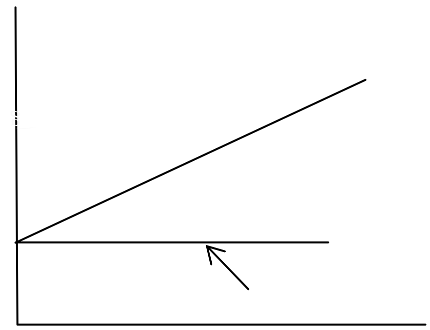

Fig 3: Dynamics of the growth rate

We now turn briefly to changes in α and β.

Recall that $g(t) = \mu(t)\beta + [1 - \mu(t)]\alpha$ and. Therefore we have (see Figs.4(a) and (b) below:

<u>Property III</u>

(a) A one-shot jump in $g_{\min}$ at date $t = 0$ shifts $g(0)$ and $g(t)$ up in the short-run (i.e. for all $t < \infty$).

> (a1) The shifted path has a gentler slope: $\dot{g}(t)$ is reduced in the short-run.
>
> (a2) The long-run growth rate is unaffected: g^* is unaffected and the shifted path still converges to g^*.

(b) A one-shot jump in g^* at $t = 0$ increases the growth rate both in the short-run as well as in the long-run.

> (b1) The shifted path is uniformly steeper.

Fig 4: The effect of a rise in (a) $g_{\min}$ and (b) g^*

Fig 5: Effect (later stages) of an increase in the laggard growth rate ($g_{\min}$)

2.2. Discussion

2.2.1. Structural Breaks?

Save in the long-run, the rate of growth is never (almost never) constant. Variations in the growth rate reflect transition dynamics. If the initial growth rate is not maximal or minimal, then the growth rate will rise over time to the maximal level. The increase in the growth rate is clearly not the result of *changes*

The second point is about the economic meaning of the term ‘structural break’. A ‘jump’ in the solution growth rate of the model is not necessarily a break. Suppose, as above, that there is a finite change in α or β. This will shift the growth path by a finite amount in the short-run – a ‘jump’. Why should that signal a ‘break’? Note that the solution path $g(t)$ is a continuously nth-differentiable function of the parameters α and β for any given t – refer to (6) below. A discrete change in these parameters would lead to a discrete change in $g(t)$ causing the path to jump or shift. It could be misleading to regard a discrete change in a parameter as a change in the structure of the model. A structural change is a regime change in a model that allows for the existence of multiple regimes. It is hasty to conclude that breaks in structure must have occurred whenever there is an observed discontinuity in the growth path or in its derivative function of time. A discrete change is roughly speaking a very large change in a very short interval of time.

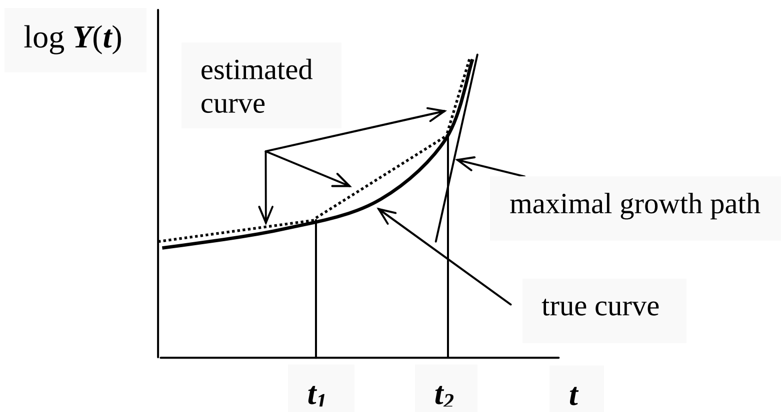

Fig 6: The Illusion of Structural Breaks at t_1 and t_2

To repeat, given the equations of the model and the levels at which the different exogenous parameters are fixed, the growth path is a continuously differentiable increasing function of time. One-shot changes in parameters may of course occur – time paths of these parameters could ‘shift’ – and that would lead to a jumps in the growth path and in rates of change. However the structure of the model may not have changed and nor may have there been a regime change.

2.2.2. Defining Consumption-driven and Investment-driven growth

What drives growth? What drives changes in growth? It is necessary to distinguish between those factors that are themselves the outcome of the model (the unknowns or endogenous variables) and those factors that are determined from outside the model and that have a role in determining the outcomes (exogenous parameters). To identify what drives growth we have to look at the exogenous parameters of the model and see how these affect GDP growth. The obvious candidates to be ‘drivers’ are (*i*) the rate of growth of autonomous consumption α and (*ii*) the rate of growth of autonomous investment β. In our model, there is convergence of the actual growth rate to the given leading rate. The laggard rate of growth represents an unstable steady state: if the initial growth rate is the minimal one it remains at that level, but a small perturbation sends the growth rate on an upward trajectory.

We may ask the following questions to identify what is driving growth.

Consider $\alpha > \beta$. Then $g(t) > \beta$ and hence *Y/I* must be rising over time. For a closed economy this means *Y/C* must be falling over time, hence *C/Y* must be rising over time. Similarly if growth is β-type, then *C*/*Y* and *C*/*I* must be falling over time. (See Property II above and the remarks preceding it. See also section 2.3 [appendix A] below on the solution).

For this rather simplistic model then the identification question is a particularly easy one to solve. It would however be misleading to believe that we could begin by *defining* growth to be consumption-driven or α-type if *C*/*I* is increasing over time. This kind of characterisation-by-

constant parameter s at the initial date does not affect $\mu(t)$ or $g(t)$. However aggregate demand $Y(t)$ gets shifted at each date t.

Here too there is possibility of an approximation error. The shift that occurs in the model is an instantaneous adjustment of demand and output. In practice this adjustment will take time. It will clearly spill over into the next period if the adjustment is slow or the unit period is short. One may therefore tend to conclude that there is a fall in the short-run rate of growth, though strictly speaking there has been no growth effect; only a level effect.

Note that the saving ratio (or the average propensity to save) will clearly rise over time as long as growth is positive since we have $S/Y = s - (\bar{C}/Y)$. This is so even when growth is consumption driven.

2.3. Solutions and Proofs of the Properties

See Appendix A

3. Full-capacity Growth: Harrod-Domar Models

Plenty of questions are left unaddressed in the above demand-determined growth model. One question is this: For how long is it possible to have positive net investment in the face of persistent excess capacity? This is especially relevant when growth itself is investment driven growth. In the case of consumption driven growth, one is led to ask how it is possible to maintain high rates of consumption growth without expanding productive capacities at sufficiently high rates over time. There must be some production-consumption consistency that puts an upper bound on the consumption growth rate.

Moreover in the pure demand model there is nothing to distinguish between autonomous consumption and autonomous investment. These are equally useful in raising current production and there is no difference as far as the future goes – since excess capacity persists by assumption.

Let us drop the assumption of everlasting excess capacities. A simple exercise is to move to the class of models that is best described as 'Harrod-Domar models' (see Domar [2] & Harrod [3]). Any positive net investment adds to capacity. In order for these new capacities to be utilized fully, there must be an appropriate increase in aggregate demand. That requires an appropriate *increase* in investment. A constant level of net inve

3.1. Generalised Harrod-Domar Model with fixed Autonomous Consumption

We consider a generalised Harrod-Domar model by incorporating autonomous consumption expenditure. Assume that in a closed economy the productivity of capital *B*, the marginal propensity to save *s* and the level of autonomous consumption $\overline{C}$ have exogenously given constant paths over time.

Let $Y = BK$ be the production relation. $(1/B)$ is the capital-output ratio. With full-capacity utilization, market clearing entails $sBK = \dot{K} + \overline{C}$. Divide by *K* and let $g_K = I/K$. Then

$$g_K = sB - (\overline{C}/K) \tag{6}$$

Thus if $K(0) > \overline{C}/sB$, then $g_K(t) > 0$ for all $t \geq 0$. If $K(0) = \overline{C}/sB$ then $K(0) = K(t)$, $g_K(t) = 0$ for all $t \geq 0$, and if $K(0) < \overline{C}/sB$ then $g_K(t) < 0$ for all $t \geq 0$.

The following phase diagram shows the dynamics (Fig. 8).

Fig 8: Dynamics of the growth paths for different levels of autonomous consumption

Assume $K(0) > \frac{\overline{C}}{sB}$. Then as $t \to \infty$, $K(t) \to \infty$ and $\frac{\overline{C}}{K(t)} \to 0$. Therefore from (6) it follows that

$g_K \to sB$ from below. From (10), $I = sBK - \overline{C}$, hence $\dot{I} = \text{sB}\,\dot{K}$, i.e.

$$g_I = sB \tag{8}$$

Hence full capacity equilibrium requires investment

This result is at sharp variance with the effect of a rise in β in the demand-based model with the Keynesian multiplier operative under condition of excess capacity. If we introduce induced investment via an accelerator, the rise in s and the resulting emergence of excess capacity would lead to a fall in investment – but that would only worsen excess capacity over time. While the rise in β raises the *equilibrium* rate of growth, the *actual* rate of growth may go down.

Next consider a one shot change in $\overline{C}$ with fixed s. This leads to what can be described as a consumption driven change. The immediate effect on the equilibrium rate of growth of capital g_K is to push it down. However, in the long run the equilibrium capital stock continues to grow at the Harrod-Domar rate β. Also note that there is no effect on the growth rate of investment g_I, either in the short or in the long run – see (8). Thus a one shot change in the autonomous consumption demand has no long run effect on the growth rates (property V).

It is interesting to see if a continuous upward drift in autonomous consumption modifies the results.

3.2 Generalized Harrod-Domar Model with growing Autonomous Consumption

Differentiating (6), with fixed β, we get

$$\dot{g}_K = -\frac{\dot{\overline{C}}}{K} + \frac{\overline{C}}{K^2}\dot{K} = \frac{\dot{\overline{C}}/\overline{C}}{K/C} + \left(\frac{\overline{C}}{K}\right)\frac{\dot{K}}{K} = -\alpha\frac{\overline{C}}{K} + \frac{\overline{C}}{K}g_K$$

$$= \frac{\overline{C}}{K}(g_K - \alpha) = (\beta - g_K)(g_K - \alpha) \qquad (9)$$

From (9), $\dot{g}_K$ is positive either when $\alpha < g_K < \beta$ or $\beta < g_K < \alpha$.

However note from (6) that it is always the case that $g_K < \beta$ for positive $\overline{C}$. Hence for growth to be positive we must have $\alpha < g_K < \beta$. Note that $\dot{g}_K$ reaches a maximum as a function of g_K at $g_K = (\alpha + \beta)/2$.

Note that for full-capacity growth $g_Y = g_K$. We drop the subscripts henceforth for brevity.

Fig 9: The dynamics of growth in generalised Harrod-Domar model

3.2.1. Properties of Growth in the Generalized Harrod-Domar model

Property VII

The economy grows, i.e. $\dot{g}(t) > 0$ provided that $\beta > \alpha$

3.2.2. Discussions

Similar to the previous model of pure demand growth, here also a rise in the average rate of exponential growth does not necessary signal a 'structural break'. The growth rate may rise on its own – except when it is too low initially – simply because it is lower than the Harrod-Domar level. In a sense the growth rate rises because initially autonomous consumption is too high relative to the capital stock.

It is crucial that the growth rate at any date must be more than α, otherwise the economy would experience a falling growth rate and hit zero growth rate in finite time. Consider a one-shot discrete jump in α to α', at some date τ . Then $\dot{g}(t)$ is lower for all $t > 0$. This means that $g(t)$ is lower for all $t > 0$ as well. If it so happens that $g(\tau) < \alpha'$, the growth rate starts falling and will eventually become negative. If $g(\tau) > \alpha'$, then the long-run rate of growth remains unaffected.

In this model, growth in the short run and may be also in the long run, is adversely affected by an increase in the rate of growth of autonomous consumption. If the jump in α is drastic, that may result in a perpetual fall in the growth rate and make it negative. Otherwise, for small up-thrusts in α, the long-run rate of growth will be maintained. This offers an interesting perspective on the growth process. Provided that α is raised by small amounts over time long run consumption growth would increase in the long-run, though the short run growth rate takes a hit.

Thus the rate of growth of autonomous consumption acts as a threshold for growth to be positive.

This result is at sharp variance with the model of pure demand determined growth. In that model a rise in α

average growth rate was 3.5 percent. During

The findings of these studies have been culminated into a recent debate: Is the recent surge in Indian growth rate ‘consumption driven’ or ‘investment driven’? However, mostly the discussion has taken the form of informal observations through the lens of Indian macro data with occasional employment of econometric methods. Moreover, there has been inadequate focus on the *composition* of aggregate demand and its drivers. There is hardly any study that uses an analytical framework to study the implications for growth of distinguishing between rising investment and rising consumption demands. Of *(i)* when $\alpha > \beta$, i.e. $\theta < 0$, as $t \to \infty$,

Fig 10 (a): The effect of changes in α on the growth rate

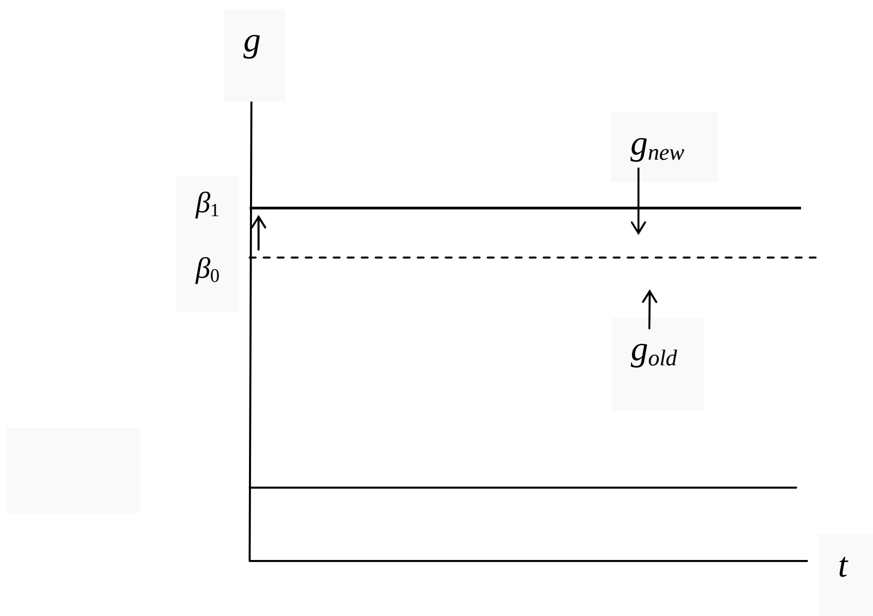

Fig 10 (b): The effect of changes in β on the growth rate

Property IV

We focus on g_C, the rate of growth of 'total' consumption C (autonomous plus induced). We know how the two components are growing – the autonomous at the rate α and the induced at the rate $g(t)$.

Differentiating the consumption function, $C = \overline{C} + cY$, we get

—
—
—
—

e know that

( ) ( ) ( ) (13)

Fig 11: The effect of rise in α (left panel) and β (right panel) on the growth path

The quantum of discrete change in α is important. After the one shot rise in α at date τ, $g(\tau)$ will

From the right hand panel of the above diagram, it is seen that $g(t)$ will continue to increase if after the increase in α, the economy is to the right of point D (the point of degeneration). Otherwise $g(t)$ will fall continuously and ev

Fig 13: Behaviour of the ratio I/C for different α

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