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A vendor managed inventory scheme for supply chain coordination under multiple heterogeneous retailers

by

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ABSRACT

This paper addresses the problem of supply chain coordination in the context of Vendor Managed Inventory (VMI) involving one supplier selling to multiple downstream heterogeneous retailers. The VMI contract considers a penalty scheme wherein, the retailers impose a per unit penalty cost on the supplier for exceeding the shipment from a pre-determined

VMI differs from traditional inventory management system in the sense that the inventory replenishment decision at the retailers’ premises is taken by the supplier instead of the retailers [6]. VMI attained prominence after the partnership between Wal-Mart and Procter & Gamble became successful in 1985 [7]. A common form of VMI contract considers a penalty scheme, where the supplier is charged a penalty for every extra unit replenished to the retailers that exceed a pre-defined limit ([5], [8], [9], [10], and [11]).

There have been several studies trying to address the issue of supply chain coordination through VMI. Studies like [5], [10], [12] and [13] have obtained centralized solution for the supply chain. Chakraborty et al. [11]–Tc -0.0e14(gm(6867(m4(a4mu6s)ltI)7(l(e)6(dc)-10(e14(gm(ti6Tw 2.00bt)-16( )613.9c

(iv) Transportation costs have not been considered separately and will be included in the ordering costs

(v) Supplier doesn't hold any inventory

2.4. Development of the Mathematical Model

Each of the '*N*' retailers when acting independently place orders according to their respective economic order quantities (EOQ) which is given by

$$Q_i = \sqrt{\frac{2D_i C_{Oi}}{C_{1i}}} \tag{1}$$

while their respective cost is given as: $B_{1i} = \frac{D_i}{Q_i} C_{Oi} + \frac{Q_i}{2} C_{1i} = \sqrt{2D_i C_{Oi} C_{1i}}$ (2)

Under VMI, the supplier has to decide on the optimal replenishment quantities for the retailers. He decides on a base replenishment cycle denoted by 'τ' and ships the retailers after every '$M_i.\tau$' time where M_i's are the integers such that for at least one i, M_i =1. Whenever the supplier replenishes anything that exceeds the upper limit z_i, he pays a penalty P_i to the retailer 'i'. The

$$p_i = \frac{x_i\left(q_i - z_i\right)^2}{2q_i}$$

$$y_i = \begin{cases} 1 & if \;\; q_i > z_i \\ 0 & otherwise \end{cases}$$

The supplier's objective is to minimize his costs post VMI implementation so that none of the retailers are made worse off. Hence the supplier's optimization problem is given as:

$$Min \quad \frac{C_S}{\tau} + \sum_{i=1}^{N} y_i p_i \tag{7}$$

S.T.

i. $\frac{C_{Oi}}{M_i \tau} + \frac{D_i M_i \tau C_{1i}}{2} - y_i p_i \le \frac{D_i}{Q_i} C_{Oi} + \frac{Q_i}{2} C_{1i} \quad \forall \; i$

ii. $D_i M_i \tau \ge y_i z_i \quad \forall \; i$

iii. $z_i \ge \left(1 - y_i\right) D_i M_i \tau \quad \forall \; i$

iv. $M_i \ge 1 \quad \forall \; i$

v. $y_i \in [0,1] \quad \forall \; i$

vi. $\tau > 0, z_i > 0 \quad \forall \; i$

Constraint (i) is the individual rationality constraint for each of the retailers indicating that a retailer will not participate in the collaborative agreement if it makes him worse off as compared to the initial case when he was ordering according to his own EOQ. Constraints (ii) and (iii) indicate that the penalty will be paid to the i^{th} retailer only if the supplier's shipment exceeds the di2(t)-2(i)c o b

3.1. JELS model with ERI

In JELS models, there is a central decision maker who aims for global optimal solution for the supply chain. Under JELS model with ERI, the supplier produces Q_s units in every cycle, and replenishes q_i units to retailer i in every such cycle i.e. $Q_s = \sum_i q_i$.

Also, $\frac{Q_s}{D} = \frac{q_i}{D_i} \quad \forall \ i$ [Equal replenishment cycle]

The total supply chain cost (to be minimized) can then be written as:-

$$TSC = \frac{D}{Q_s} C_s + \frac{D}{Q_s} \sum_i C_{Oi} + \frac{1}{2} \sum_i q_i h_i = \frac{D}{Q_s} C_s + \frac{D}{Q_s} \sum_i C_{Oi} + \frac{1}{2} \sum_i Q_s \frac{D_i}{D} h_i \qquad \text{using} \quad \frac{Q_s}{D} = \frac{q_i}{D_i}$$

Differentiating above with respect to Q_s gives the following optimal values:

$$Q_S^* = \sqrt{\frac{2\, D^2 \left(C_s + \sum_i C_{0i} \right)}{\sum_i D_i C_{1i}}} \text{ and } q_i = \frac{D_i}{\sum_i D_i} Q_S^*$$

Second order condition gives $\frac{2D}{Q_S^3} C_s + \frac{2D}{Q_S^3} \sum_i C_{Oi} > 0$ for $Q_S > 0$ and hence Q_s^* gives the optimal solution.

3.2. JELS model with URI

Under such a setting the supplier doesn't restrict himself to a common replenishment cycle and selects replenishment period given by $M_i\tau$ where M_i are integers and τ is the base replenishment cycle. The objective function can be written as given below:

of penalty scheme and the same models will be used for comparison purpose with our proposed model.

4. Equivalence of the proposed model and JELS model with URI

In this section we establish the equivalence between our proposed model and JELS model under unequal replenishment intervals.

The constraint (i) in the proposed model can be rearranged as follows:

$y_i p_i \geq \frac{C_{Oi}}{M_i \tau} + \frac{D_i M_i \tau C_{1i}}{2} - \frac{D_i C_{Oi}}{Q_i} - \frac{Q_i C_{1i}}{2} \quad \forall \ i$ which acts as the side payment from the supplier to each of the retailers to compensate them for carrying extra inventory. The same constraint will be binding in nature, as the supplier will always prefer to pay the retailers as less as possible since any payment more than the least value of $y_i.p_i$ would reduce the gains of the supplier.

Thus we can write: $y_i p_i = \frac{C_{0i}}{M_i \tau} + \frac{D_i M_i \tau C_{1i}}{2} - \frac{D_i}{Q_i} C_{Oi} - \frac{Q_i}{2} C_{1i} \quad \forall \ i$

$$\frac{\quad_{0i}}{} \qquad \frac{_i \quad _i \quad _{1i}}{2} \qquad \frac{_i}{} \qquad \frac{_i}{2} \ _1$$

Unlike other models, the proposed model is such that every retailer (irrespective of its size) remains in the VMI partnership willingly.

5.1.2. Impact on the individual retailer's cost by the variation in its ordering cost

The data used for the sensitivity analysis is as shown in table 5.

Table 5: Data for the sensitivity analysis 5.1.2.

Retailer data				Supplier data
Retailer	D_i (Demand)	C_{1i} (Inventory holding cost)	x_i (per unit penalty)	C_s (Setup cost)
R1	400	0.8	0.5	120
R2	1000	0.8	0.5	
R3	3000	0.8	0.5	
R4	8000	0.8	0.5	

We aim to study the impact on individual retailer's cost when its ordering cost is varied from 5 to 100 through 20 instances. While we vary the ordering cost for one of the retailers, the ordering cost for all the other retailers is kept at 40. Specifically, we study the impact on the smallest (R1) and the biggest (R4) retailer by performing the analysis independently on both. We measure the impact by the percentage deviation in the retailer's cost from the ideal scenario under the EOQ. The results for the sensitivity analysis can be summarized as follows :-

6. Conclusion

In this paper, we develop a model for determining the optimal replenishment policy in the context of single supplier multiple retailer scenario under VMI. It has been found that the proposed model outperforms the existing models in the literature. The contributions of the paper can be summarized in the following points:-

1) We provide the optimal replenishment policy in the context of single supplier–multiple retailer scenario under VMI.
2) The JELS model is known to provide the optimal replenishment policy in the context of single supplier – single retailer scenario. In this paper, we show that the same is also true for single supplier – multiple retailer scenario. Further, we also establish the equivalence of the proposed model with the JELS model.
3) In a multiple retailer situation two types of replenishment policy arise, equal and unequal replenishment interval. Through our proposed model, we reaffirm the point that URI policies being more generalized outperform the ERI policies.
4) Through the sensitivity analysis we show that the proposed model is much more robust than the other models. We find (for the proposed model) that the individual retailers remain unaffected by various factors such as presence of the bigger retailers and the change in the ordering cost.

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